Proceedings of the Fifth IASTED International Conference
MODELLING, SIMULATION, AND OPTIMIZATION
August 29-31, 2005, Oranjestad, Aruba

RECENT ADVANCES IN ADAPTIVE SYSTEM IDENTIFICATION BASED ON SPARSE SIMULATION-BASED FREQUENCY-DOMAIN DATA

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ABSTRACT
Accurate simulation and modeling of complex multi-port LTI systems can be computationally very expensive and resource-demanding. Adaptive system identification techniques, that combine adaptive sampling and modeling algorithms, can be used to minimize the number of costly data samples, and to build accurate broadband models in a limited amount of time. This paper gives a survey of several recent advances in the world of deterministic system identification, based on sparse and costly data.
KEY WORDS
Adaptive techniques, System Identification, Rational Approximation.

1 Introduction

Linear Time Invariant (LTI) systems, such as electronic devices, are often characterized in the frequency-domain using rational functions. Efficient domain-specific simulators are available to model the complex system behaviour at discrete frequencies, often at a high computational cost. Multiple costly simulations or virtual measurements must be performed, to characterize a system over a broad frequency range. Adaptive system identification techniques, that combine adaptive sampling and adaptive modeling algorithms, can be used to reduce the computational cost and to improve the model accuracy [1]. This approach allows the automatical selection of an appropriate model complexity and sample distribution. It avoids overmodeling and undermodeling, as well as oversampling and undersampling. No prior knowledge of the system is assumed.

2 Rational Modeling

A rational analytic model $R(s)$ is defined as a quotient of two polynomials $N(s)$ and $D(s)$.

$$R(s) = \frac{N(s)}{D(s)} = \frac{\sum_{n=0}^{N} N_n s^n}{\sum_{d=0}^{D} D_d s^d} \quad s = j2\pi f \qquad (1)$$

where N and D represent the order of numerator and denominator respectively, and N_n and D_d the polynomial coefficients. The rational function provides an approximation of the spectral response of the system over the interval [f_{min},f_{max}]. Since there are $N + D + 1$ unknown coefficients (D_0 can be chosen arbitrarily, e.g. $D_0 = 1$), a set of $K + 1 = N + D + 1$ samples $(s_k, H(s_k))$ are required to completely determine $R(s)$. $R(s)$ is then a curve passing through the values $H(s_k)$ at the complex frequencies s_k for $k = 0, 1, ..., K$. We will assume that such a rational function exists, and that it has no unattainable points. To calculate the coefficients of the model, several fitting techniques were presented in the past, each having it's own strengths and weaknesses [2].

For ease of understanding, the Rational Linear Least Squares (RLLS) technique is used to parameterize the model, however the ideas presented here are general enough to be applied to several other least-squares fitting techniques as well.

The identification problem is linearized by multiplying left hand side and right hand side of equation (1) with the denominator polynomial, and equating R(s) to H(s) for all frequencies s_k.

$$\left(\sum_{d=0}^{D} D_d s^d\right) H(s) = \sum_{n=0}^{N} N_n s^n \qquad (2)$$

At a given frequency point, we get (D_0=1)

$$A_k x = b_k \qquad (3)$$

where

$$A_k = \left[\begin{array}{ccccccc} 1 & ... & s_k^N & -s_k H(s_k) & ... & -s_k^D H(s_k) \end{array}\right] \qquad (4)$$

$$x = \left[\begin{array}{ccccccc} N_0 & ... & N_N & D_1 & ... & D_d \end{array}\right]^T \qquad (5)$$

$$b_k = [H(s_k)] \qquad (6)$$

A direct solution with real coefficients can be found, by writing out (3) for all frequencies s_k, and solving the following set of linear equations

$$\left[\begin{array}{c} \Re(A) \\ \Im(A) \end{array}\right] x = \left[\begin{array}{c} \Re(b) \\ \Im(b) \end{array}\right] \qquad (7)$$

This approach -combined with an appropriate frequency scaling- can be used to build rational models. However,

systems which are modeled over a very broad frequency range, and highly dynamic systems can't be modeled with sufficient accuracy, because of numerical problems. The Vandermonde-structure of A makes the system severely ill-conditioned. To resolve the numerical issues, rational splines can be used. Instead of splitting the frequency range of interest in an ad hoc number of subranges, and model them all separately, adaptive techniques can be used during the modeling process to determine an optimal knot location, based on the numerical conditioning of the problem [3].

3 Adaptive Sampling Algorithm

The flowchart of the adaptive algorithm is shown in Figure 1. It consists of an adaptive modeling loop, and an adaptive sample selection loop [4].

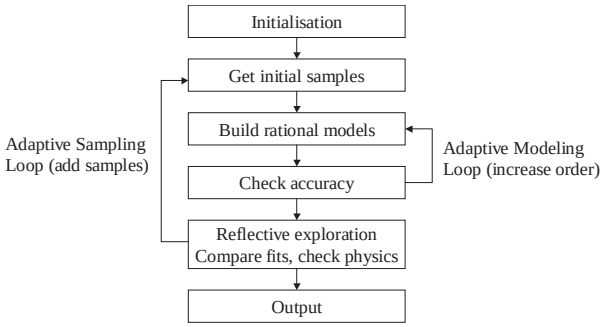


Figure 1. Flowchart of the Adaptive Sampling and Modeling Algorithm

The goal is to minimize the error of the fitting model according to the following error criterium

$$dB(|R(j\omega) - H(j\omega)|) < threshold \qquad (8)$$

The algorithm starts with 4 samples equidistantly spaced over a certain frequency range of interest. Depending on the number of available data samples, multiple rational models are built with different order of numerator and denominator, exploiting all degrees of freedom. All rational fitting models are evaluated in the data points, and compared against one another. If the error between the model, evaluated in the selected sample points and the simulated data samples exceeds a certain threshold, the model is rejected, and the model's complexity is increased. All models with different order of numerator and denominator are ranked, and the 2 best models (i.e. with lowest overall error) are retained. The difference between these 2 models is called the estimated fitting error, and new samples should be chosen in such way, that the maximum estimated fitting error is minimized.

Note that the estimated fitting error is always an estimation of the real error, as this would only be known after performing a lot of computationally expensive verification simulations. Although the estimated fitting error provides a good measure to determine the frequency where the uncertainty of the model is the largest, it can sometimes cause the algorithm to converge prematurely. A good way to increase the reliability of the method, is to combine this estimated fitting error with a heuristic engine. Each time new models are generated, the algorithm checks multiple heuristic rules, and terminates when they are all satisfied.

Such rules, called *reflective functions* [5], compare

- correspondance of the phase
- correspondance of the magnitude
- correspondance of the Euclidean distance in the complex plane

between

- fitting model and simulated data samples
- fitting models, based on an overdetermined set of equations (approximants)
- fitting models, calculated when all interpolation conditions are satisfied (interpolants)
- fitting models, based on a different set of support samples
- fitting models, based on a subset of selected support samples
- fitting models, based on neighbouring and overlapping frequency ranges

while detecting

- passivity violations
- other unphysical effects

The major disadvantage of the technique is that the reliability of convergence-detection is highly dependent on the set of rules, and the appropriate choice of thresholds. In [6], an alternative strategy was introduced, which is based on the "survival-of-the-fittest" principle of genetic algorithms. An explicit convergence function was established. However if the rational models require a large number of poles, the convergence function becomes unreliable, and the algorithm converges prematurely. To resolve these issues, the technology was refined and more robust fitting techniques were applied. Also, the method was extended to multi-port systems [7].

4 Frequency Derivatives

In some cases, it's possible to obtain t^{th} order frequency derivatives $(s_k, H^{(t)}(s_k))$ from the simulator. Frequency derivatives are scaled moments (coefficients of the Taylor

$$H^{(t)}(s_k)\sum_{d=0}^{D} D_d {s_k}^d = \sum_{n=t}^{N} N_n {s_k}^{n-t}\frac{n!}{(n-t)!} - \sum_{m=1}^{t}\sum_{d=m}^{D}\begin{pmatrix} t \\ m \end{pmatrix} H^{(t-m)}(s_k) D_d {s_k}^{d-m}\frac{d!}{(d-m)!} \quad (9)$$

series at a given expansion point), which can often be simulated at a significantly lower computational cost than data samples. Taking them into account can significantly reduce the overall simulation cost, since they provide additional information to the modeling process. Some Finite Element Method (FEM) simulators can take advantage of this property.

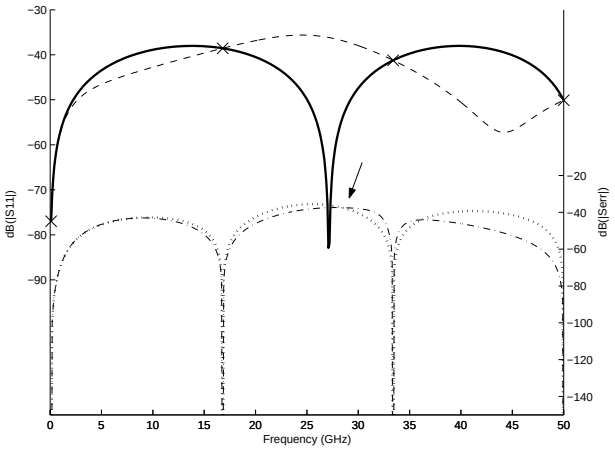
To solve the identification problem, it is desired to satisfy $R^{(t)}(s_k) = H^{(t)}(s_k)$ for all $k = 0, 1, ..., K$ and $t = 0, 1, ..., T$ [8]. Again, we will assume that such a rational function exists, and that it has no unattainable points.

When frequency derivatives of the data are available at the discrete frequencies s_k, (2) can be generalized by taking them into account. The coefficients N_n and D_d of the rational fitting model now satisfy equation (9). All derivatives are relative to s. The set of equations at all frequencies s_k and for all derivatives t, can be written in a similar matrix form as equation (7).

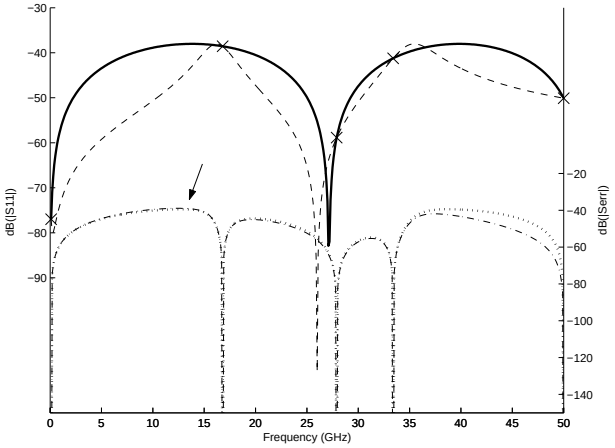
5 Example

The reflection coefficients S_{11} of a 2-port Coplanar Waveguide (CPW) are modeled with the adaptive sampling technique over the frequency range [0.1 GHz - 50 GHz]. All data samples are simulated with the planar full-wave electro-magnetic simulator Agilent EEsof Momentum [9]. The desired model accuracy of the S-parameters is -60dB or better, which corresponds to a maximal error on the magnitude of 0.001.

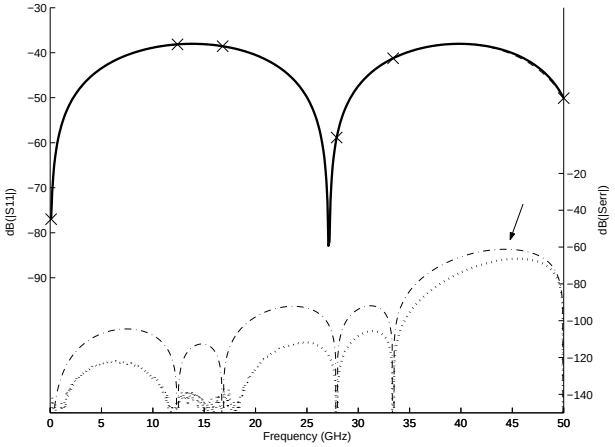
Figure 2 a-d shows the consecutive steps of the algorithm. The component is modeled using the default RLLS, i.e. without making use of frequency derivatives. The algorithm initially starts with 4 samples, equidistantly spaced over the frequency range of interest, and builds several interpolation models. The "best" model is shown as a dashed line. In each iteration of the algorithm, new samples are selected at frequencies where the estimated error (the difference between 2 rational approximants, based on the same set of support samples) is maximal. These new sample locations are marked with an arrow. Based on the extra data points, new rational models are built and evaluated, and the estimated error function is updated. The iterative process is repeated until the error is below a predefined accuracy threshold. In this example, 7 support samples are automatically selected, to obtain the required precision. The real error (difference between the model and very densely selected verification data), which is also shown in the figures, is usually not known during the modeling process. In this example, it is given only for illustration to the reader. Note that there is often a strong correlation between the estimated error and real error.



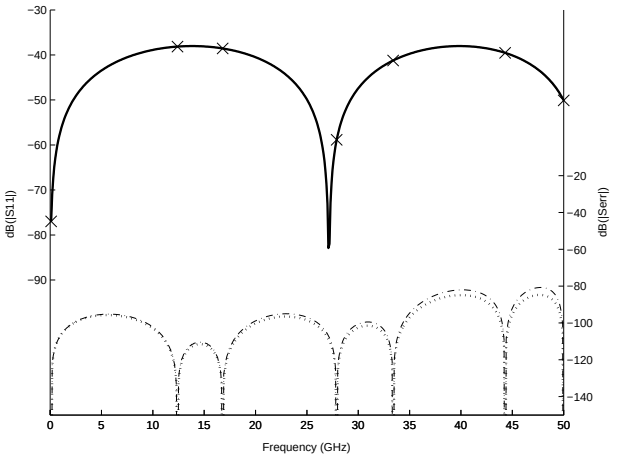
a. 4 samples selected



b. 5 samples selected

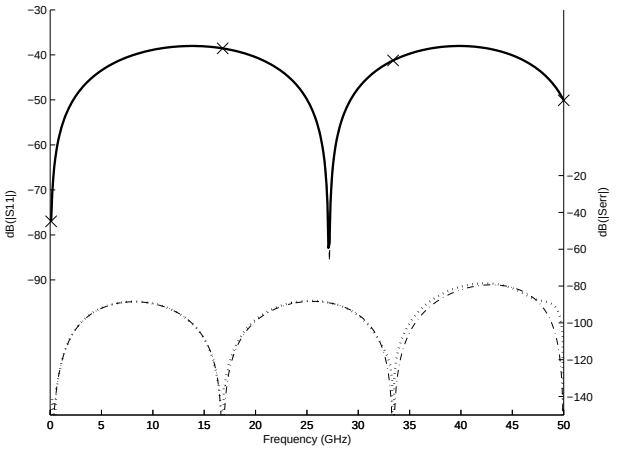


c. 6 samples selected



d. 7 samples selected

Figure 2. Adaptive sampling and modeling of CPW. Magnitude of best rational model (dashed line) and reference data (full line) shown on left axis. Estimated error (dash-dotted line) and real error (dotted line) shown on the right axis. No frequency derivatives are used.



a. 4 samples selected + frequency derivatives

Figure 3. Adaptive sampling and modeling of CPW. Magnitude of best rational model (dashed line) and reference data (full line) shown on left axis. Estimated error (dash-dotted line) and real error (dotted line) shown on the right axis. Data samples and 1st frequency derivatives are used.

In Figure 3 a, the first frequency derivatives are also used in the modeling process. This additional information is exploited by the algorithm, and now it only needs 4 samples (+ first order derivatives) to find an accurate model.

6 Conclusions

Adaptive frequency sampling and modeling algorithms are particularly useful when the computational cost of simulating data samples is very high. Adaptively built models have a much higher accuracy at a much lower computational cost than traditional approaches based on equidistant sampling and linear interpolation. When frequency derivatives of the data are used in the modeling process, the total simulation cost can be reduced even further.

7 Acknowledgements

This work was supported by the Fund for Scientific Research Flanders.

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