Adaptive Building of Accurate and Stable PEEC Models for EMC Applications

Giulio Antonini
EMC Laboratory
Dipartimento di Ingegneria Elettrica
Università degli Studi di L'Aquila
Poggio di Roio, 67040, L'Aquila, Italy
Email: antonini@ing.univaq.it

Dirk Deschrijver, Tom Dhaene
Computer Modeling and Simulation (COMS)
Department of Mathematics and Computer Science
University of Antwerp,
Middelheimlaan 1, 2020 Antwerp, Belgium
Email: {dirk.deschrijver,tom.dhaene}@ua.ac.be

***Abstract*— A new adaptive technique is presented for building accurate and stable Partial Element Equivalent Circuit (PEEC) models over a wide frequency range. Rational models are generated for impedances corresponding to partial inductances and coefficients of potential over a frequency range of interest, based on a limited number of samples. Delay extraction is applied in order to keep the order of the rational models as low as possible. The adaptive algorithm doesn't require any a-priori knowledge of the dynamics of the system to select an appropriate sample distribution and an appropriate model complexity.**

I. Introduction

Transient analysis of electromagnetic compatibility (EMC) problems can be carried out using either integral or differential equation (IE or DE) based methods. With the increasing need to analyze wide-band, time-varying and non linear problems, a robust time-domain solution technique becomes increasingly urgent. Time domain integral equation (TDIE) based methods have a number of advantages over other techniques: 1) they require only the discretization of conductors and dielectrics; 2) as time domain technique they return broadband information in a single run and 3) they allow an easy treatment of time-varying and non linear problems. The main drawback of TDIE methods is related to the issue of their instabilities. Over the past few years the computational efficiency of marching-on-in-time (MOT) schemes for solving TDIEs has been significantly improved by the use of fast solution schemes such as the plane-wave time domain ad hierarchical fast Fourier transform (FFT) methods [1], [2]. Nonetheless the stability is still an open issue.

Among the integral equation based technique the Partial Element Equivalent Circuit [3], [4] is well suited for mixed circuit and electromagnetic (EM) problems as it provides a circuit interpretation of the electric field integral equation (EFIE). Thus, it is becoming more and more popular among EMC engineers for its capability to handle complex problems involving EM and circuits problems. Like other TDIE techniques it may suffer from time domain instabilities. In the recent past many authors have worked on solving the problem of instability separately from the issue of accuracy. In the framework of the PEEC it is clear that they cannot be separate. In [5] it is shown that the accuracy in the computation of partial elements, namely partial inductances and coefficients of potential, at very high frequency, may be cause of instabilities; in that work a macromodel is proposed to ensure accuracy over a wide frequency range, thus leading to better stability properties of the overall time domain model. In [6] an improved formulation of the PEEC method is presented that is based on a delay extraction technique and a rational modeling of impedances corresponding to partial elements describing the electric and magnetic field couplings; the generated macromodels are accurate over a wide frequency range and, at the same time, allow to improve the stability of the resulting (Lp, P, R, τ)PEEC model. The main issue in applying the above mentioned technique is the need to sample the partial elements in the frequency domain before macromodeling can be applied. Frequency domain computation of partial elements can be time consuming when the problem at hand is electrically large. In this work we present an adaptive frequency sampling algorithm (AFS) for fast and accurate PEEC modeling. Basically, a preliminary delay extraction is applied, and, secondly rational models are generated by using adaptive frequency sampling. The order of rational models is kept low by the delay extraction technique. The examples presented confirm the efficiency of the proposed algorithm in ensuring accuracy and in improving the stability properties of the PEEC models at a reduced computational cost.

II. Frequency dependent partial elements

As stated in the previous Section, accuracy and stability of integral equation based methods are not separate issues. In the PEEC framework stability is strictly related to the accuracy of partial elements computation. Thus, in the present Section, firstly we review the partial elements, partial inductances and coefficients of potential, describing the magnetic field and electric field couplings.

A. Partial inductances

The magnetic field coupling between volume cells m and n is described in terms of the complex partial inductance

$$L_{p,mn}(s) = \frac{\mu}{4\pi a_m a_n} \int_{v_m} \int_{v_n} \frac{e^{-s|\boldsymbol{r}_m - \boldsymbol{r}_n|/c_0}}{|\boldsymbol{r}_m - \boldsymbol{r}_n|} dv_m dv_n \tag{1}$$

where c_0 is the free space speed of light, a_m and a_n the cross section of volume cells m and n. In time domain PEEC modeling it is common practice to assume a center to center approximation for the exponential term which can be taken out of the integral yielding

$$L_{p,mn}(s) = e^{-s\tau_{mn}^{L}} L_{p,mn}^{st} \tag{2}$$

where $\tau_{mn}^{L} = R/c_0$ is the center to center fly-time between cells m and n, R is the center to center distance between the cells. The standard delay extraction implies reduced, and thus acceptable, loss of accuracy for electrically small structures but may cause large errors in the case of electrically large objects whose dimensions exceed λ_{min}, corresponding to the maximum frequency of interest. The direct rational approximation of partial inductances and coefficients of potential over a broad frequency band is a difficult task because of the exponential term into (2). It firstly causes the time delay of the electromagnetic field to propagate from the source to the observation point which results into fast variations in phase; secondly the dispersion [7] which results into slow variations in magnitude. Such kind of problem is usually mitigated by using delay extraction. In the PEEC framework all the electric as well as magnetic field couplings are established in air since dielectric polarization currents are modeled locally by excess capacitances. This fact allows an easy and reasonable estimate of the delay as the center to center time of flight, as in (2). Nevertheless the approximation (2) may be significantly wrong for electrically large structures because of the effect of dispersion which still affects the delayless part. It results into a poor modeling in the active ($freq \leq f_{max}$) frequency range. Furthermore, approximation (2) surely is not accurate in the extended ($f_{max} < freq \leq 20 \times f_{max}$) frequency range, which significantly impacts the stability properties of the overall PEEC model [8]. Thus, a better and more accurate model is required in both the active and extended ranges.

As previously stated the time delay for the electromagnetic field to propagate from cell n to cell m can be approximated as $\tau_{mn}^{L} = R/c_0$ where R is the center to center distance. This task allows to write the complex impedance $Z_{L,mn}(s)$ as

$$Z_{L,mn}(s) = sL_{p,mn}(s) = Z_{L,mn}^{dl}(s)\, e^{-s\tau_{mn}^{L}} \tag{3}$$

where the delayless impedance $Z_{L,mn}^{dl} = sL_{p,mn}(s)\, e^{s\tau_{mn}^{L}}$ is still frequency dependent and takes dispersive phenomena into account at very high frequencies. The branch voltage induced on the volume cell m due to the current flowing in the volume cell n reads

$$V_{b,mn}(s) = Z_{L,mn}^{dl}(s)\, I_{L,n}(s)\, e^{-s\tau_{mn}^{L}} \tag{4}$$

Typically the computation of its time domain counterpart can be performed by

a) standard convolution techniques;
b) recursive convolution techniques via rational approximation of $Z_{L,mn}^{dl}(s)$;

In the following the circuit synthesis of $Z_{L,mn}^{dl}(s)$ is performed along with its rational approximation yielding

$$\begin{aligned} Z_{L,mn}^{dl}(s) &= d_{mn}^{L} + s e_{mn}^{L} + \sum_{k=1}^{N_p^r} \frac{Res_k^r}{s - p_k^r} \\ &+ \sum_{k=1}^{N_p^c} \left(\frac{Res_k^c}{s - p_k^c} + \frac{\overline{Res}_k^c}{s - \overline{p}_k^c} \right) \end{aligned} \tag{5}$$

B. Coefficients of potential

The electric field coupling is described by complex coefficient of potential. Considering two cells, m and n the mutual coefficient of potential is

$$P_{mn}(s) = \frac{1}{4\pi\varepsilon S_m S_n} \int_{S_m} \int_{S_n} \frac{e^{-s|\boldsymbol{r}_m - \boldsymbol{r}_n|/c_0}}{|\boldsymbol{r}_m - \boldsymbol{r}_n|} dS_m dS_n \tag{6}$$

where S_m and S_n represent the area of surface cells m and n. The impedance describing mutual electric field coupling is

$$Z_{C,mn}(s) = \frac{P_{mn}(s)}{s} \tag{7}$$

The center to center delay can be extracted thus providing

$$Z_{C,mn}(s) = Z^{dl}_{C,mn}(s)\, e^{-s\tau^{C}_{mn}} \tag{8}$$

where $Z^{dl}_{C,mn}(s)$ is the delayless impedance

$$Z^{dl}_{C,mn}(s) = Z_{C,mn}(s)\, e^{s\tau^{C}_{mn}} \tag{9}$$

The pole-residue representation of $Z_{C,mn}(s)$ leads to

$$\begin{aligned} Z^{dl}_{C,mn}(s) &= d^{C}_{mn} + se^{C}_{mn} + \sum_{k=1}^{N_p^r} \frac{Res^r_k}{s-p^r_k} \\ &+ \sum_{k=1}^{N_p^c} \left(\frac{Res^c_k}{s-p^c_k} + \frac{\overline{Res}^c_k}{s-\overline{p}^c_k} \right) \end{aligned} \tag{10}$$

The main drawback of the previously outlined approach resides in the necessity of preliminarily computing the partial elements over a wide frequency range so to capture the damping in their magnitude. Usually the computation is performed in the extended range $f_{max} = 20\ f_a$ where f_a is the maximum frequency at which the models are accurate being the $\lambda/20$ criterion satisfied (active range).

Due to $sinx/x$ nature of the damping [9] the behavior of partial elements magnitude is quite smooth and an adaptive frequency sampling technique is likely applied.

III. Adaptive Sampling Algorithm

Robust frequency domain fitting methods [10], [11] can be applied to characterize the impedance data ($s_w, Z^{dl}_{L,mn}(s_w)$ or $Z^{dl}_{C,mn}(s_w)$) by a rational improper transfer function $R(s_w)$, which is of the form (5) or (10).

The goal of the algorithm is to identify the unknown system parameters d, e, Res_k, and p_k, in such way that the accuracy of fitting model is bounded by the following error function

$$E_{RMS} = \sqrt{\frac{\sum_{w=1}^{W} |R(s_w) - Z(s_w)|^2}{W}} < \tau \tag{11}$$

For this particular application, the threshold τ is chosen to be 10^{-4}.

In order to satisfy this requirement, the order of the model (denoted by N_p) should be sufficiently high. Also, the sampling density of the structure should be chosen in such way that all spectral dynamics of the system are sufficiently sampled, especially where the impedances are changing more rapidly.

In the most common situations, one can resort to a very dense uniform sample distribution. Although this method can be useful when the data is cheap to obtain, it can be computationally expensive and resource demanding when the simulation of data samples is costly. Reducing the spectral density of the data samples can be an option when the data behaves smoothly, however a higher accuracy of the model is obtained if the samples are selected more optimally with adaptive sampling algorithms [12]. These techniques automatically select a quasi-optimal sample distribution, and an appropriate model complexity, without requiring any a-priori knowledge of the system.

The flow chart of the algorithm is shown in Figure 1. It consists of an adaptive modeling loop, and an adaptive sample selection loop.

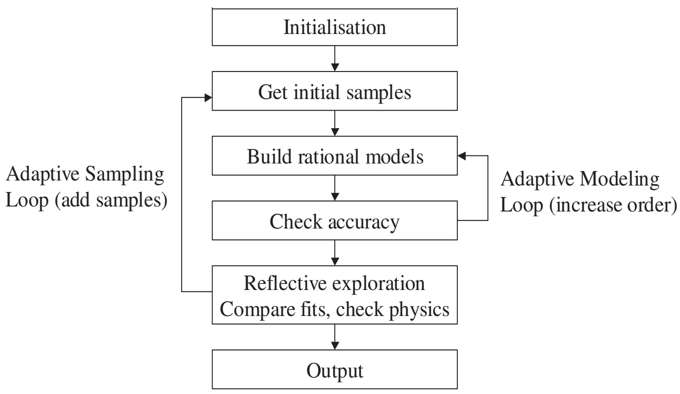


Fig. 1. Flowchart of the adaptive sampling and modeling algorithm.

The algorithm starts with 4 samples equidistantly spaced over a certain frequency range of interest. Depending on the number of available data samples, multiple rational models are built with different order of numerator and denominator, exploiting all degrees of freedom. All rational fitting models are evaluated in the data points, and compared against one another. If the error between the model, evaluated in the selected sample points and the simulated data samples exceeds a certain threshold, the model is rejected, and the model's complexity is increased. All models with different order of numerator and denominator are ranked, and the 2 best models with lowest overall error (say R_1 and R_2) are retained. The difference between these 2 models is called the estimated fitting error

$$E_{RMS}^{est} = \sqrt{\frac{\sum_{w=1}^{W} |R_1(s_w) - R_2(s_w)|^2}{W}} \quad (12)$$

and new samples should be chosen in such way, that the maximum estimated fitting error is minimized. For impedances, one can select data samples e.g. at the frequency s_w where

$$\max_w \frac{|R_1(s_w) - R_2(s_w)|}{|R_1(s_w)|} \quad (13)$$

Note that the estimated fitting error is always an estimation of the real error, as this would only be known after performing a lot of computationally expensive verification simulations. Although the estimated fitting error provides a good measure to determine the frequency where the uncertainty of the model is maximal, it can sometimes cause the algorithm to converge prematurely. A good way to increase the reliability of the method, is to combine this estimated fitting error with a heuristic engine. Each time new models are generated, the algorithm checks the heuristic rules, and terminates when they are all satisfied.

Such rules, called reflective functions [13], compare e.g.

- Correspondance of the phase
- Correspondance of the magnitude
- Correspondance of the Euclidean distance in the complex plane

between

- Fitting model and simulated data samples
- Fitting models, calculated from overdetermined set of equations (approximants)
- Fitting models, calculated when all interpolation conditions are satisfied (interpolants)
- Fitting models, based on a different set of support samples
- Fitting models, based on a subset of selected support samples
- Fitting models, based on neighbouring and overlapping frequency ranges

while detecting passivity violations and other unphysical effects.

Unfortunately, it can be hard to define a reliable set of reflective functions, since it requires a lot of experience and know-how. Therefore, one could also resort to a GA-inspired algorithm [14] which is easy to implement, and gives reliable results as well. The major advantage of this approach, is that an explicit measure for the quality (fitness) of a model, and the convergence of the algorithm can be established. Moreover, such method extends gracefully to multi-port systems.

IV. Numerical results

A. Near field coupling

In the first test two cells, α and β, are considered, with dimensions $l_\alpha = 1.5$ mm, $l_\beta = 1.5$ mm, $w_\alpha = 1.5$ mm and $w_\beta = 0.15$ mm. This ensures that the cell dimensions match the requirement $max(l_\alpha, l_\beta, w_\alpha, w_\beta) \leq \lambda_{min}/20$ at the active frequency $f_a = 10$ GHz. The maximum frequency at which the models need to be accurate is $f_{max} = 20\ f_a = 200$ GHz. They are touching, as shown in Fig. 2. Near field couplings have an important impact on stability and, therefore, the corresponding partial elements need an accurate computation in both the active and extended frequency ranges.

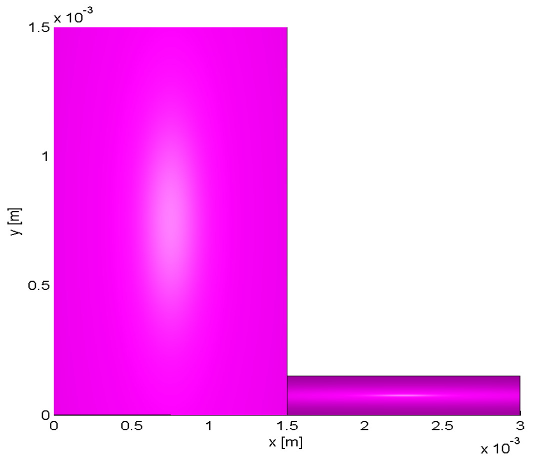


Fig. 2. Two touching cells (example IV-A).

The self and mutual coefficients of potential have been computed by means of different methods summarized in Table I. In the first test, the cells are coplanar and are touching along one edge. The reference results have been obtained by a frequency dependent Gauss-Legendre integration of order 10 over the surface (S-FD). Further, the center to center approximation has been also assumed and the integral has been computed by using a frequency independent surface integration (S-cc). Fig. 3 shows the mutual partial inductance $L_{P,12}$ evaluated by means of the S-FD and S-cc techniques: it is clearly seen that the center to center approximation 2 leads to poor results into the active and extended frequency ranges.

Then, the delayless mutual impedance $Z_{L,\alpha\beta}^{dl}(s) = sL_{p,\alpha\beta}(s)$ has been computed and fitted applying the

TABLE I

METHODS OF COMPUTATION FOR COEFFICIENTS OF POTENTIAL

Method	Description
S-FD	Frequency dependent gaussian surface integration
S-cc	Frequency independent gaussian surface integration with center to center delay approximation

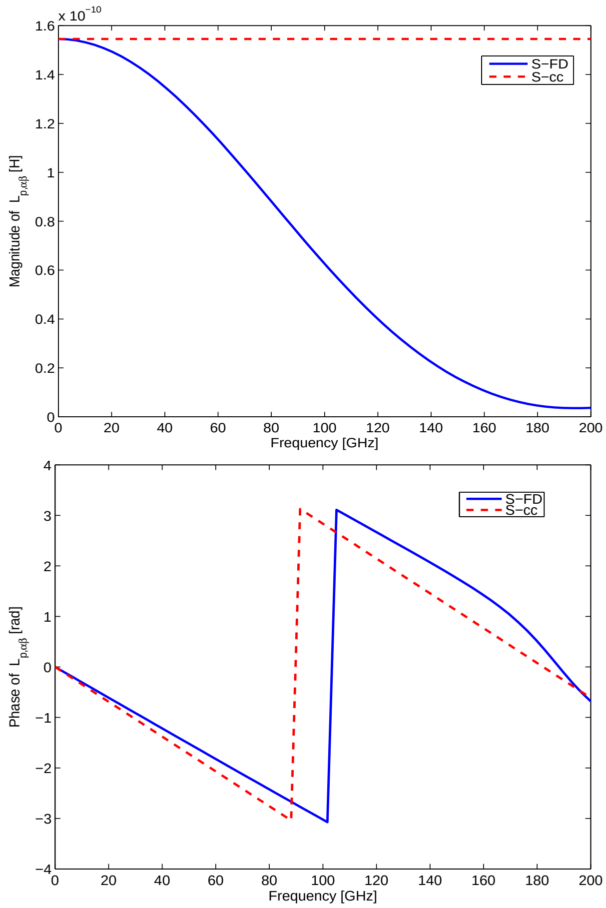


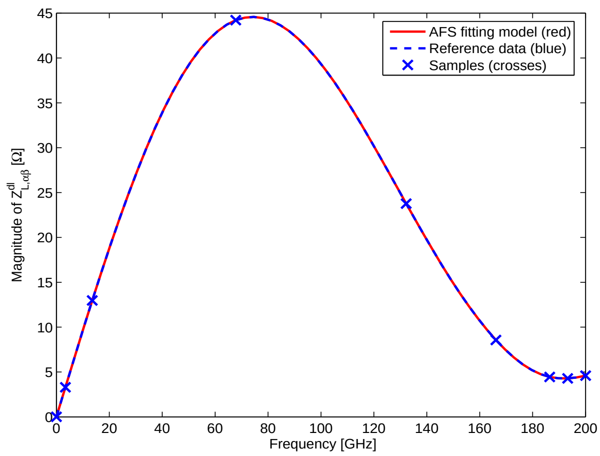
Fig. 3. Mutual partial inductance $L_{P,12}$ (example IV-A). Top: magnitude; bottom: phase.

AFS algorithm requiring only 9 frequency samples. The results are shown in Fig. 4.

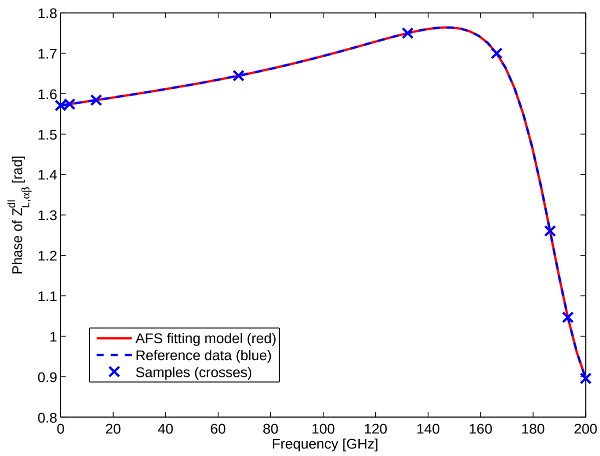
The adaptive frequency sample procedure allows to build a rational model which is accurate up to 200 GHz at a low computational cost. It is to be pointed out that the damping of the mutual impedance is properly modeled, thus preserving accuracy and improving stability.

B. Far field coupling

In the second test two square cells α and β are considered, with dimensions $l_\alpha = l_\beta = 1.5$ mm,



(a) Magnitude.



(b) Phase.

Fig. 4. Mutual impedance $Z^{dl}_{L,\alpha\beta}(s)$ (example IV-A). a) Magnitude. b) Phase.

$w_\alpha = w_\beta = 1.5$ mm; they are $4\ \lambda_{min}$ far apart, as shown in Fig. 5. Again, the delay extraction is applied along with the AFS algorithm to the delayless mutual impedance $Z^{dl}_{L,\alpha\beta}(s) = sL_{p,\alpha\beta}(s)$. In this case 10 frequency samples have been selected to build a rational model accurate up to 200 GHz, as Fig. 6 confirms. Also in this case the rational model is able to capture the physical damping of the mutual impedance by using a very limited number of frequency samples.

V. CONCLUSIONS

In this paper we present an innovative approach for efficiently building accurate and stable PEEC models over a wide frequency range. The proposed method combines three techniques: 1) delay extraction so that low order rational models can be adopted; 2) adaptive frequency sampling is used to build rational models of impedances describing the magnetic field and electric

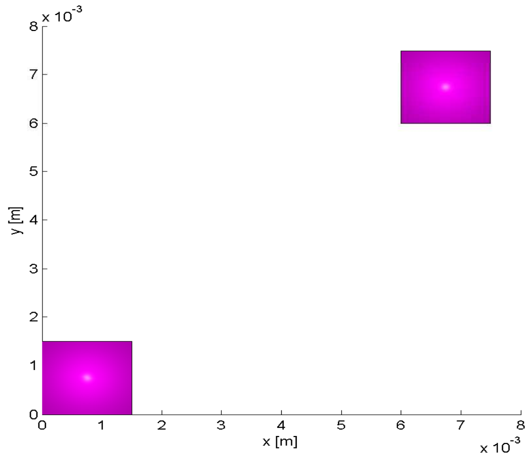
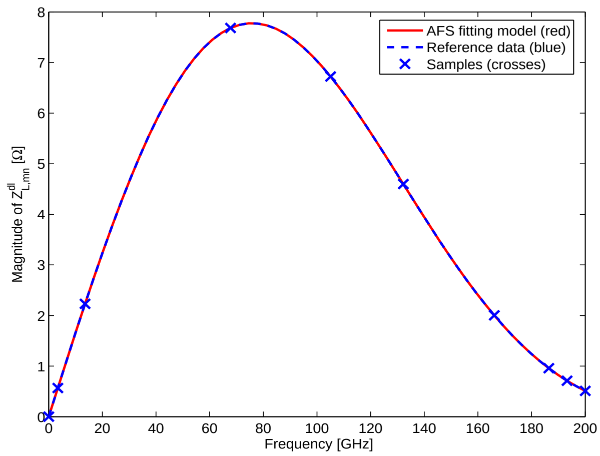


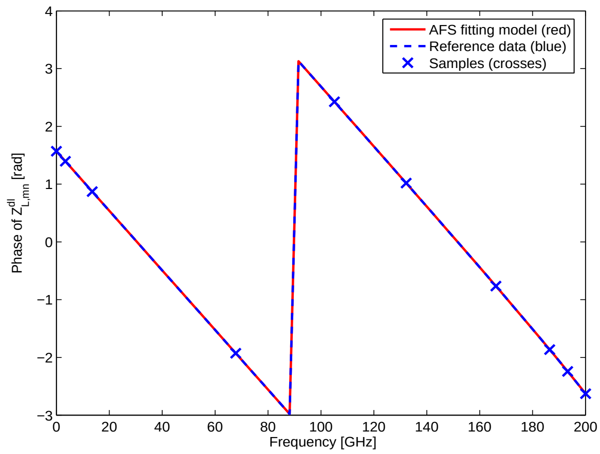
Fig. 5. Two non touching cells (4 λ_{min} far apart, example IV-B).

field couplings in the framework of PEEC modeling. The examples presented confirm the robustness of the proposed method in ensuring accuracy in both the active and extended frequency ranges and in improving the stability of the resulting PEEC models as a consequence of the proper modeling of impedances damping.

Future work will involve generating stable time domain PEEC models by using the proposed approach, analyzing passivity, estimating computational costs.



(a) Magnitude.



(b) Phase.

Fig. 6. Mutual impedance $Z^{dl}_{L,\alpha\beta}\left(s\right)$ (example IV-B). a) Magnitude. b) Phase.

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