Single-Input-Single-Output Passive Macromodeling via Positive Fractions Vector Fitting

Luciano De Tommasi[1-2], Dirk Deschrijver[3] and Tom Dhaene[3-1]
[1]University of Antwerp, Department of Mathematics and Computer Science – Middelheimlaan 1, B-2020 Antwerp, Belgium
[2]NXP Semiconductors – High Tech Campus 37, PostBox WY4-01, NL-5656AE Eindhoven, Netherlands
[3]Ghent University - IBBT, Department of Information Technology (INTEC), Gaston Crommenlaan 8, B-9050 Ghent, Belgium
luciano.detommasi@ua.ac.be, dirk.deschrijver@ugent.be, tom.dhaene@ugent.be

Abstract

This paper introduces a constrained Vector Fitting algorithm which can directly identify a passive driving point function (impedance or admittance) from frequency domain data. The proposed Positive Fractions Vector Fitting (PFVF) algorithm formulates the residue identification step as a convex programming problem, while the pole identification step follows the unaltered standard Vector Fitting procedure. A further extension to multi-input-multi-output functions is possible and is under investigation.

Introduction

The design of modern electronic systems requires accurate modeling of electrical interconnects in order to address several signal integrity issues. Lumped macromodels, which enable fast time domain simulations through recursive convolutions, can be identified using standard transfer function or state space realization techniques (e.g. [1]). Interconnects and electronic packaging are passive structures and have to be described through passive macromodels as well, since a non-passive model may lead to unstable transient simulations when connected with some termination networks [2-4].

In this paper, we deal with the identification problem of a passive macromodel (1) from frequency domain data samples $\{\omega_k, \tilde{Z}(j\omega_k)\}_{k=1\ldots K}$, which are obtained from measurements or full-wave simulations.

$$Z(s) = a_0 + \sum_{n=1}^{N_r} \frac{a_n}{s-p_n} + \sum_{n=N_r+1}^{N_r+N_c} \left(\frac{a_n}{s-p_n} + \frac{a_n^*}{s-p_n^*} \right) \approx \tilde{Z}(s) \,. \quad (1)$$

The Vector Fitting (VF) algorithm [1],[5] is well known as a valid approach to pursue the identification of poles $\{p_n\}$ and residues $\{a_n\}$ of (1), but it can not guarantee that the identified transfer function is a positive real function (i.e. it represents a passive system), even if the original data $\{\omega_k, \tilde{Z}(j\omega_k)\}_{k=1\ldots K}$ come from a passive system. Therefore, passivity must be checked after the identification of $Z(s)$. If any violations are found, the macromodel is then corrected via perturbation-based approaches [2-4]. The main drawback of these passivity enforcement schemes is that they may diverge, so destroying the accurate approximation found by VF.

Some recent efforts oriented to the direct identification of passive macromodels [6-7] show that there is interest to overcome these limitations of the perturbation-based passivity enforcement schemes. A direct identification of a passive state space realization is pursued in [6] by using the following *positive real lemma*, which provides a necessary and sufficient condition for the passivity of a system.

Positive real lemma: Let a linear and time-invariant system admit the state space realization *{A,B,C,D}* and the transfer function $H(s) = C(sI-A)^{-1}B + D$. If there exists a matrix $W=W^T$ such that:

$$\begin{bmatrix} -A^T W - WA & -WB + C^T \\ -B^T W + C & D + D^T \end{bmatrix} \geq 0 \,, \qquad W \geq 0 \quad (2)$$

then *H(s)* is a positive real function. Moreover, if *H(s)* is positive real then there exists a *W* such that (2) are satisfied.

According to [6], the most straightforward way to solve the identification problem (1) including the positive real lemma constraint (and assuming that the poles $\{p_n\}$ are known), is via semidefinite programming.

The main issue with the application of the positive real lemma is that the optimization algorithm has to handle all additional variables of *W*. Therefore, due to the increased complexity of the optimization problem, the identification is limited to quite low orders. In order to reduce the number of variables in the optimization step, a custom mathematical formulation was pursued in [6]. Nevertheless, because of the difficult formulation and lack of a public domain implementation, such approach has not become a standard approach in the macromodeling community yet.

We aim to investigate an alternative approach, easier to understand and implement than [6] and which does not introduces more variables than the transfer function residues $\{a_n\}$. Of course, this approach also has some potential drawbacks: the macromodel identification will be based on a sufficient but not necessary condition for the passivity, and therefore may lead to sub-optimal solutions.

Passive Macromodeling via Convex Programming

Similarly to the formulation [6], the PFVF algorithm is based on convex optimization and assumes that a suitable set of stable poles $\{p_n\}$ (i.e. poles in the left half of the complex plane) has already been identified for the transfer function under study (1). The standard unconstrained least squares identification step of VF, including pole flipping, is used in the first step to estimate the poles. In the second step, the residues $\{a_n\}$ of (1) are calculated by minimizing the error between the original data $\{\omega_k, \tilde{Z}(j\omega_k)\}_{k=1\ldots K}$ and $Z(j\omega_k)$, under the passivity constraint.

Differently from [6], the passivity constraint is not imposed by requiring that the positive real lemma is satisfied. The proposed PFVF procedure assumes that each simple fraction corresponding to a real pole and each couple of simple fractions corresponding to a complex conjugate pair, are positive real functions and that the residue a_0 is non-negative [4],[8]. This is a sufficient condition to guarantee that the entire transfer function is positive real, being the sum of positive real terms, and can be easily translated in a set of

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convex constraints involving the residues $\{a_n\}$, once that poles $\{p_n\}$ are assumed to be known. Clearly, the condition is not necessary too, since the nonnegativeness of $\mathrm{Re}\{Z(j\omega)\}$ does not imply the nonnegativeness of the real part of all its partial fractions.

Respectively, for a real pole p_r<0 and for a complex conjugate pair $\sigma_c \pm j\omega_c$ (where σ_c<0), the positive real condition becomes:

$$\mathrm{Re}\left\{\frac{a}{j\omega - p_r}\right\} \geq 0 \ \text{ for any } \omega \Leftrightarrow a \geq 0 \tag{3}$$

$$\mathrm{Re}\left\{\frac{\alpha + j\beta}{j\omega - \sigma_c - j\omega_c} + \frac{\alpha - j\beta}{j\omega - \sigma_c + j\omega_c}\right\} \geq 0 \ \text{ for any } \omega \Leftrightarrow \Leftrightarrow \begin{cases} -(\alpha\sigma_c + \beta\omega_c) \geq 0 \\ -(\alpha\sigma_c - \beta\omega_c) \geq 0 \end{cases} \tag{4}$$

In both cases, since poles are known quantities, the constraints are linear inequalities.

The PFVF algorithm performs the residue identification step minimizing the error function:

$$\sum_{k=1}^{K} |Z(j\omega_k) - \widetilde{Z}(j\omega_k)|^2, \tag{5}$$

where

$$Z(j\omega) = a_0 + \sum_{n=1}^{N_r} \frac{a_n}{j\omega - p_n} + \sum_{n=1}^{N_c} \left(\frac{b_n j\omega + c_n}{-\omega^2 + d_n j\omega + e_n} \right). \tag{6}$$

The unknowns $\{a_n, b_n, c_n\}$ are computed while enforcing the constraints:

$$a_n \geq 0 \quad \text{for } n = 0..N_r, \tag{7}$$

$$\begin{cases} b_n d_n - c_n \geq 0 \\ c_n \geq 0 \end{cases} \quad \text{for } n = 1..N_c. \tag{8}$$

The constraints (8) are equivalent to (4).

To solve the problem of minimizing (5) under the constraints (7) and (8), we used CVX (version 1.1 beta), a package for specifying and solving convex programs [9].

Example

As a test case we consider a lossy transmission line of length 10 cm, with the following p.u.l. parameters:

R = 1.4 Ω/m, **L** = 0.63 μH/m, **G** = 50 mS/m, **C** = 110 pF/m.

The *Z*-parameters have been analytically computed and then the proposed PFVF algorithm was applied to fit Z_{11} using 30 poles. Some results are shown in Fig. 1 (magnitude) and Fig. 2 (phase angle) and compared using the standard VF identification (same number of poles and iterations). In both cases unitary weights have been applied.

The constraints (3) and (4) have been checked for both the models identified using the VF algorithm and the proposed PFVF algorithm. The former does violate the constraints, possibly resulting in a non-passive model. On the other hand, the latter does not violate any constraints therefore guaranteeing the passivity.

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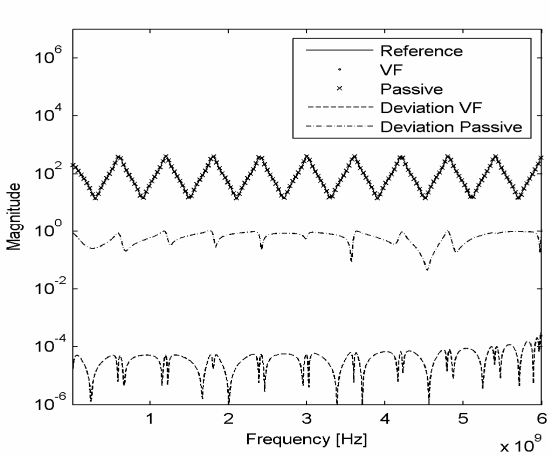

Fig. 1 Magnitude of the reference function $Z_{11}(f)$ compared to those of identified functions via VF and PFVF algorithms.

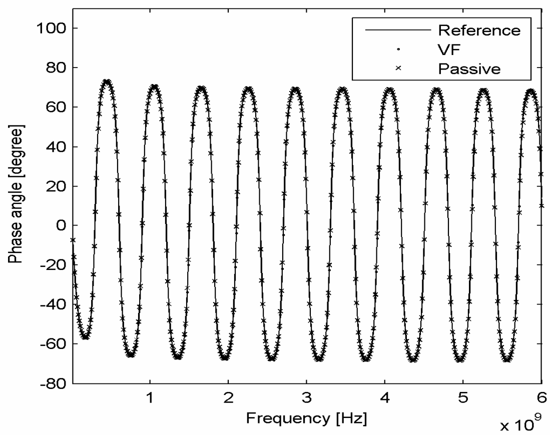

Fig. 2 Phase angle of the reference function $Z_{11}(f)$ compared to those of identified functions via VF and PFVF algorithms.

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