# Fast Parametric Macromodeling of Frequency Responses Using Parameter Derivatives

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***Abstract*—This letter presents a generalization of the multivariate Vector Fitting technique that includes parameter derivatives in the macromodeling process. The computational cost to simulate partial derivatives in terms of the parameters is often substantially lower than the simulation cost of additional data samples. An example shows that the inclusion of derivatives can be useful to reduce the required amount of data samples, while preserving the accuracy of the results.**

***Index Terms*—Least-squares, parametric macromodels, rational functions, surface approximation, vector fitting.**

## I. INTRODUCTION

INCREASING integration levels in microwave devices and higher signal speeds require accurate modeling of previously neglected interconnection effects during circuit and system simulations. Accurate prediction of these effects is fundamental for successful design and involves the solution of large systems of equations which are often prohibitively expensive to solve. For real-time design space exploration and fast optimization, there is a significant need for accurate broadband parametric macromodels that approximate the frequency-domain behavior of a system in terms of several design variables by a rational analytic function.

Recently, a robust multivariate extension of the Orthonormal Vector Fitting technique was introduced in [1]. This method combines the use of an iterative least squares estimator and orthonormal rational functions which are based on a prescribed set of poles. It was shown that the method is able to compute accurate parametric macromodels, based on parameterized frequency responses which exhibit a highly dynamic behavior.

This letter generalizes this technique, to include parameter derivatives in the modeling process. Parameter derivatives provide additional information about the underlying system and can often be simulated at a significantly lower computational cost than additional samples [2]–[5]. The presented algorithm can exploit this information to compute a parametric macromodel in a reduced amount of time. An example illustrates the increased efficiency that can be obtained. For ease of notation, the new macromodeling algorithm is only described for bivariate systems. Of course, the full multivariate formulation can be derived in a completely similar way.

Manuscript received May 30, 2008; revised August 18, 2008. Current version published December 04, 2008. This work was supported by the Research Foundation Flanders (FWO).

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Digital Object Identifier 10.1109/LMWC.2008.2007687

## II. PARAMETRIC MACROMODEL

It was proposed in [1] to represent the parametric macromodel as the ratio of a bivariate numerator and denominator

$$R(s,g) = \frac{N(s,g)}{D(s,g)} = \frac{\sum_{p=0}^{P}\sum_{v=0}^{V} c_{pv}\phi_p(s)\varphi_v(g)}{\sum_{p=0}^{P}\sum_{v=0}^{V} \tilde{c}_{pv}\phi_p(s)\varphi_v(g)} \tag{1}$$

where $s = j\omega$ is the complex frequency variable and $g$ is a real design variable. The maximum order of the corresponding basis functions $\phi_p(s)$ and $\varphi_v(g)$ is denoted by $P$ and $V$ respectively. Based on a set of data samples $\{(s,g)_k, H(s,g)_k\}_{k=0}^{K}$, the algorithm pursues the identification of the model coefficients $c_{pv}$ and $\tilde{c}_{pv}$ of numerator and denominator in (1).

## III. ITERATIVE ALGORITHM

In the first iteration step of the algorithm $(t = 0)$, Levi's cost function [6] is minimized to obtain an initial guess of the coefficients. In successive iteration steps $(t = 1, \ldots, T)$, the Sanathanan–Koerner cost function is minimized [7], [8], which uses the inverse of the previously estimated denominator

$$\left(D^{(t-1)}(s,g)_k\right)^{-1} = w^{(t)}(s,g)_k \tag{2}$$

as an explicit weight factor to the least-squares equations. All details about this procedure are well reported in [1]. This letter proposes a generalized Sobolev space cost function, that takes derivatives of the parameter variables into account [9], [10]

$$E = \sum_{k=0}^{K}\left(\sum_{q=0}^{Q_s}\left|e_{1,q}^{(t)}\right|^2 + \sum_{q=1}^{Q_g}\left|e_{2,q}^{(t)}\right|^2\right)\Bigg|_{(s,g)=(s,g)_k}. \tag{3}$$

This cost function $E$ in (3) is minimized for $c_{pv}^{(t)}$, $\tilde{c}_{pv}^{(t)}$ where

$$e_{1,q}^{(t)} = \frac{\partial^q}{\partial s^q}\left(\frac{N^{(t)}(s,g)}{D^{(t-1)}(s,g)} - \frac{D^{(t)}(s,g)}{D^{(t-1)}(s,g)}H(s,g)\right) \tag{4}$$

$$e_{2,q}^{(t)} = \frac{\partial^q}{\partial g^q}\left(\frac{N^{(t)}(s,g)}{D^{(t-1)}(s,g)} - \frac{D^{(t)}(s,g)}{D^{(t-1)}(s,g)}H(s,g)\right) \tag{5}$$

provided that $Q_s$ and $Q_g$ represent the order of the partial derivatives in $s$ and $g$ respectively. Although it is possible to include cross derivatives in a similar way, this procedure is not described for the sake of simplicity. In order to avoid the trivial

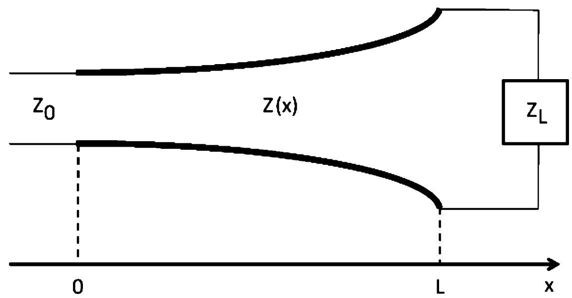


Fig. 1. Exponential tapered transmission line [16].

null solution of the least squares problem (3), the constant term $\tilde{c}_{00}^{(t)}$ of the denominator in (1) is set to unity. Each equation is then split in its real and imaginary parts, to ensure that the model coefficients $c_{pv}^{(t)}$, $\tilde{c}_{pv}^{(t)}$ are real. It is also noted that the numerical accuracy of the results can be improved by scaling each column to unity length [11]. To ensure that the poles of the parametric macromodel are located in the left-half plane, some stability conditions can be verified if needed [12].

## IV. Inclusion Parameter Derivatives

The following formula $\Psi_q$ describes a notational shorthand for Leibniz identity, which generalizes the product rule for expressing higher-order derivatives of products of functions

$$\Psi_q\left(f(x), g(x), x\right) = \sum_{m=0}^{q} \binom{q}{m} \frac{\partial^m}{\partial x^m} f(x) \frac{\partial^{q-m}}{\partial x^{q-m}} g(x) \quad (8)$$

provided that $f(x)$ and $g(x)$ are arbitrary functions that depend on the dummy variable $x$. An application of the Leibniz identity to (4) and (5) yields the following equations:

$$\begin{aligned} e_{1,q}^{(t)} = &\Psi_q\left(w^{(t)}(s,g), N^{(t)}(s,g), s\right) \\ &- \Psi_q\left(w^{(t)}(s,g), D^{(t)}(s,g)H(s,g), s\right) \quad (6) \\ e_{2,q}^{(t)} = &\Psi_q\left(w^{(t)}(s,g), N^{(t)}(s,g), g\right) \\ &- \Psi_q\left(w^{(t)}(s,g), D^{(t)}(s,g)H(s,g), g\right). \quad (7) \end{aligned}$$

$g(x)$ represents the produt of two functions, then the Leibniz identity is applied in a recursive fashion.

It is noted that the derivatives of $N^{(t)}(s,g)$, $D^{(t)}(s,g)$ and $w^{(t)}(s,g)$ with respect to the parameters $s$ and $g$ are also needed to solve (8). The derivatives of $N^{(t)}(s,g)$ and $D^{(t)}(s,g)$ (as defined in (1)) are directly based on the derivatives of the basis functions $\phi_p(s)$ and $\varphi_v(g)$, which are reported in the next section. The weighting function $w^{(t)}(s,g)$ can be considered as a composition of the reciprocal function and $D^{(t-1)}(s,g)$, such that the derivatives can be computed using the Faà di Bruno formula. This formula decomposes the compound expression in terms of Bell polynomials [13].

## V. Generalization of the Basis Functions

### A. Frequency-Dependent Basis Functions

A set of partial fractions $\phi_p(s,\vec{a})$ is chosen, which are based on a prescribed set of stable poles $\vec{a} = \{-a_p\}_{p=1}^{P}$, provided that $\phi_0(s) = 1$. These poles are selected as complex conjugate pairs with small real parts and the imaginary parts linearly spaced over the frequency range of interest [11]. A linear combination of two partial fractions is formed to ensure that both $\phi_p(s,\vec{a})$ and $\phi_{p+1}(s,\vec{a})$ are real-valued functions

$$\phi_p(s,\vec{a}) = (s+a_p)^{-1} + (s+a_{p+1})^{-1} \quad (9)$$
$$\phi_{p+1}(s,\vec{a}) = j(s+a_p)^{-1} - j(s+a_{p+1})^{-1}. \quad (10)$$

If the following auxiliary function is defined as:

$$\Theta_q(s,a_p) = \frac{\partial^q}{\partial s^q}(s+a_p)^{-1} = (-1)^q q!(s+a_p)^{-(q+1)} \quad (11)$$

then the derivatives with respect to $s$ are given by

$$\frac{\partial^q}{\partial s^q}\phi_p(s,\vec{a}) = \Theta_q(s,a_p) + \Theta_q(s,a_{p+1}) \quad (12)$$
$$\frac{\partial^q}{\partial s^q}\phi_{p+1}(s,\vec{a}) = j\Theta_q(s,a_p) - j\Theta_q(s,a_{p+1}). \quad (13)$$

### B. Parameter-Dependent Basis Functions

The parameter-dependent basis functions $\varphi_v(g,\vec{b})$ are also rational functions, which are chosen in partial fraction form as a function of $jg$. These basis functions are based on a prescribed set of starting poles $\vec{b} = \{-b_v\}_{v=1}^{V}$, which are chosen as complex pairs with small real parts of opposite sign and imaginary parts linearly spaced over the parameter range of interest, provided that $\varphi_0(g) = 1$. A linear combination of two partial fractions is formed to ensure that $\varphi_v(g,\vec{b})$ and $\varphi_{v+1}(g,\vec{b})$ are strictly real functions by construction [14]

$$\varphi_v(g,\vec{b}) = (jg+b_v)^{-1} - (jg-(b_v)^*)^{-1} \quad (14)$$
$$\varphi_{v+1}(g,\vec{b}) = j(jg+b_v)^{-1} + j\left(jg-(b_v)^*\right)^{-1}. \quad (15)$$

If the following auxiliary function is defined as

$$\Lambda_q(g,b_v) = \frac{\partial^q}{\partial g^q}(jg+b_v)^{-1} = (-j)^q q!(jg+b_v)^{-(q+1)} \quad (16)$$

then the derivatives with respect to $g$ are given by

$$\frac{\partial^q}{\partial g^q}\varphi_v(g,\vec{b}) = \Lambda_q(g,b_v) - \Lambda_q\left(g,-(b_v)^*\right) \quad (17)$$
$$\frac{\partial^q}{\partial g^q}\varphi_{v+1}(g,\vec{b}) = j\Lambda_q(g,b_v) + j\Lambda_q\left(g,-(b_v)^*\right). \quad (18)$$

## VI. Example: Tapered Transmission Line

The proposed technique is used to model the reflection coefficient $S_{11}$ of a lossless exponential tapered transmission line [15], [16] that is terminated with a matched load, as shown in Fig. 1, where $Z_0 = 50\ \Omega$ and $Z_L = 300\ \Omega$ represent the reference impedance and the load impedance, respectively. The relative dielectric constant $\epsilon_r$ is chosen equal to 2.

A bivariate parametric macromodel is computed as a function of varying line length $L \in [1\ \mathrm{cm} - 10\ \mathrm{cm}]$ over the frequency range [1 kHz–8 GHz]. The desired model accuracy is set to −60 dB, which corresponds to three significant digits. The number of poles of the macromodel is set to 10 for the length parameter and 18 for the frequency. If no parameter derivatives are

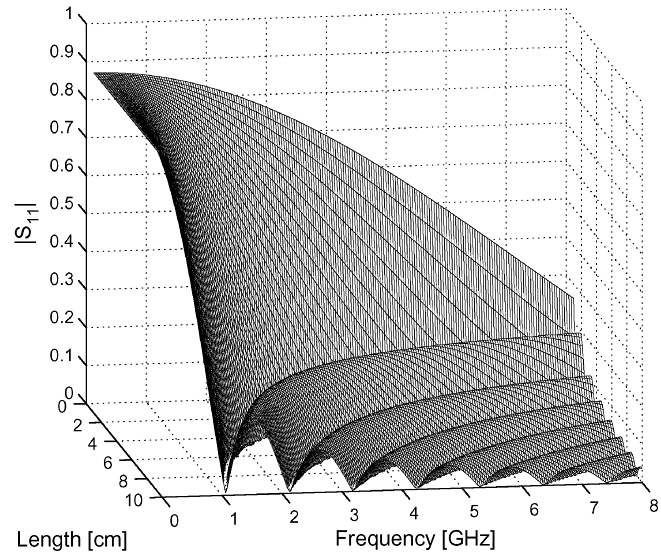


Fig. 2. Reflection coefficient $S_{11}$ of macromodel using first order derivatives.

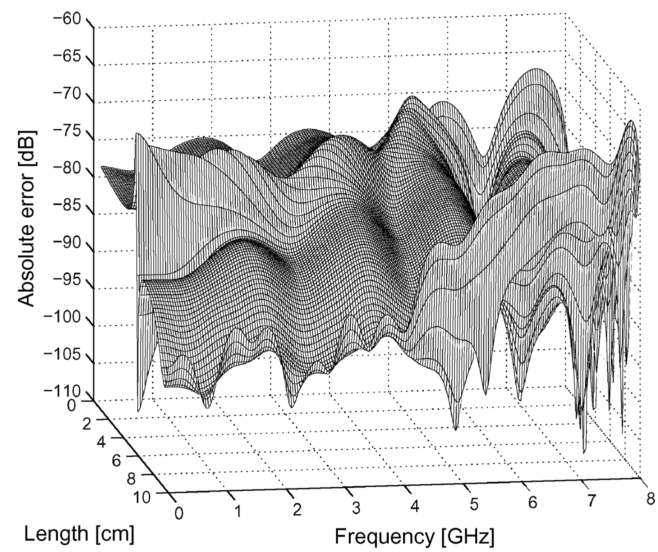


Fig. 3. Absolute error of macromodel using first order derivatives.

used, at least $46 \times 50$ uniformly distributed data samples are required to obtain the desired accuracy of the macromodel. When the first-order derivatives for length and frequency parameters are also included, only $17 \times 18$ data samples are needed to obtain a similar accuracy.

The response of the parametric macromodel is evaluated over a dense set of $80 \times 200$ data samples, as shown in Fig. 2. Fig. 3 confirms that an overall good agreement is observed between the macromodel and the set of validation samples.

## VII. Conclusion

A generalization of the multivariate Vector Fitting technique [1] that includes parameter derivatives is presented, to compute parametric macromodels from frequency response data. Partial derivatives can often be obtained at a significantly lower computational cost than additional samples and yield useful information in the fitting process. Numerical results show that the inclusion of parameter derivatives can reduce the required amount of samples, while preserving the model accuracy.

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