Efficient Algorithm for Passivity Enforcement of S-Parameter-Based Macromodels

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***Abstract*—This paper presents an efficient and robust algorithm for passivity enforcement of S-parameter-based macromodels. The method computes updated values of the model residues by least squares fitting of nonpassive residuals of the scattering matrix. Several examples show that the proposed method yields accurate passive macromodels at a limited computational cost.**

***Index Terms*—Least squares fitting, macromodeling, model perturbation, passivity enforcement, vector fitting.**

I. Introduction

VECTOR fitting has become a standard approach for robust and accurate macromodeling of passive microwave systems from tabulated S-parameter data [1], [2]. A known restriction of the technique is that the computed macromodels are not guaranteed to be passive by construction. Nevertheless, passivity of the macromodel is of crucial importance since a nonpassive macromodel may lead to unstable transient simulations in an unpredictable manner. Several techniques have recently been considered to address this issue, ranging from convex optimization [3] to Nevanlinna-pick interpolation [4], semi-definite programming [5], linear or quadratic programming [6], [7], residue perturbation [8], [9], pole perturbation [10], [11], modal perturbation [12], waveform shaping [13], and others [14]–[17]. A comparative study of several passivity enforcement schemes has recently been reported in [18].

This paper introduces a robust algorithm that is able to enforce passivity to a nonpassive rational macromodel by means of an overdetermined least squares fitting algorithm. The main benefit of this approach is that it does not rely on optimization procedures, which are often numerically expensive or possibly nonconvergent. At the same time, the implementation of the proposed algorithm is simple and straightforward. Several numerical examples illustrate that the presented approach achieves an excellent tradeoff between computation time and accuracy preservation of the overall macromodel.

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II. Macromodeling

Vector fitting is an efficient macromodeling technique to compute a rational function approximation from the scattering matrix of a given linear structure [1], [2]. A direct application of the algorithm to the simulated or measured frequency response yields a stable, but potentially nonpassive macromodel that is formulated in a compact pole-residue form

$$S_{mn}(j\omega) = \sum_{p=1}^{P} \frac{c_p^{mn}}{j\omega - a_p^{mn}} + d^{mn} \tag{1}$$

where $S_{mn}(j\omega)$ represents the corresponding element on row m and column n of the scattering matrix. The poles a_p^{mn} and residues c_p^{mn} are real or come in complex conjugate pairs, while d^{mn} is a constant real term. All elements of the scattering matrix can be fitted with a common set of poles ($a_p^{mn} = a_p$) or a separate set of poles for each scattering element. A complex diagonalized state-space realization of the compound system can easily be derived, as shown in [1], [19]

$$j\omega X(j\omega) = AX(j\omega) + BU(j\omega) \tag{2}$$

$$Y(j\omega) = CX(j\omega) + DU(j\omega). \tag{3}$$

It is ensured that all the poles of the macromodel are strictly stable, such that the eigenvalues of A have negative real parts [20]. Asymptotic passivity of the macromodel is also enforced.

III. Passivity Condition Check

The definition of passivity for S-parameter-based macromodels in the frequency domain stipulates that all singular values σ of scattering matrix $S(j\omega)$ are unitary bounded [21]

$$\left(I - S^H(j\omega)S(j\omega)\right) \geq 0 \qquad \forall\omega \tag{4}$$

which leads to the following equivalent expression

$$\max_{\omega} \sigma\left(S(j\omega)\right) \leq 1 \qquad \forall\omega. \tag{5}$$

This condition can easily be verified algebraically by computing the eigenvalues of an associated Hamiltonian matrix [22]

$$H = \begin{bmatrix} A - BR^{-1}D^TC & -BR^{-1}B^T \\ C^TQ^{-1}C & -A^T + C^TDR^{-1}B^T \end{bmatrix} \tag{6}$$

where $R = D^TD - I$ and $Q = DD^T - I$. If $j\omega_k$ is an imaginary eigenvalue of H, then the corresponding frequency ω_k may denote the crossover between a passive and a nonpassive frequency band [23]. By computing the slopes of the singular value curves at the purely imaginary eigenvalues, it is possible to pinpoint the exact boundaries of a passivity violation. If all the eigenvalues of H have a nonvanishing real part, then the system is passive. Theoretical proofs about this procedure are reported in [22]. In the case of reciprocal systems with a symmetric scattering matrix, it is possible to derive a new test matrix, which is only half the size of the Hamiltonian matrix. This leads to savings in the eigenvalue computation time by a factor of eight (see [24] for details).

IV. Passivity Compensation

If the state-space model (2), (3) is found to be nonpassive by the Hamiltonian test (6), then a new passivity enforcement algorithm can be applied to compensate the violation. The presented approach iteratively updates the residues in the output matrix C_t (for $t = 0, \ldots, T$) by a simple least squares fitting procedure until all passivity violations are removed. In the first iteration step $t = 0$ of the algorithm, $C_0 = C$ in (3).

A. Nonpassive Residuals of Scattering Matrix

First, a dense set of frequencies Ω_{eval} is determined from dc up to about 20% above the highest relevant frequency. This highest relevant frequency is the maximum of the highest crossing from a nonpassive to a passive region on one hand and the maximum frequency of interest on the other hand. For each frequency ω_{eval} in the set Ω_{eval}, a singular value decomposition (SVD) of the scattering matrix is performed as follows:

$$S(j\omega_{\mathrm{eval}}) = D + C_t(j\omega_{\mathrm{eval}}I - A)^{-1}B = U\Sigma V^* \qquad (7)$$

where Σ is a positive, real-valued diagonal matrix that contains the singular values, and U and V are unitary matrices. The inversion of the $(j\omega_{\mathrm{eval}}I - A)$ in (7) is computationally fast because it is a complex diagonal matrix. It is clear that one (or several) of the singular values in Σ will exceed unity in the areas where the model is nonpassive. Therefore, a new set of violation parameters S_{viol} is constructed as follows:

$$S_{\mathrm{viol}}(j\omega_{\mathrm{eval}}) = U\Sigma_{\mathrm{viol}}V^* \qquad \forall \omega_{\mathrm{eval}} \in \Omega_{\mathrm{eval}} \qquad (8)$$

with

$$\Sigma_{\mathrm{viol}} = \Sigma\Upsilon - \Psi \qquad (9)$$

where Υ and Ψ are square diagonal matrices

$$\begin{aligned} \Upsilon|_{ii,\Sigma_{ii}\le\delta} &= 0 \quad \Upsilon|_{ii,\Sigma_{ii}>\delta} = 1 \\ \Psi|_{ii,\Sigma_{ii}\le\delta} &= 0 \quad \Psi|_{ii,\Sigma_{ii}>\delta} = \delta. \end{aligned} \qquad (10)$$

The value of δ is a predefined tolerance parameter that is chosen slightly smaller than 1 in practice (such as, e.g., 0.999).

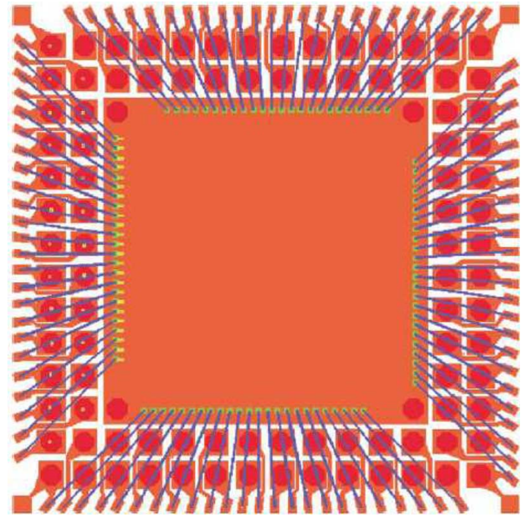

Fig. 1. BGA package: top view of the structure.

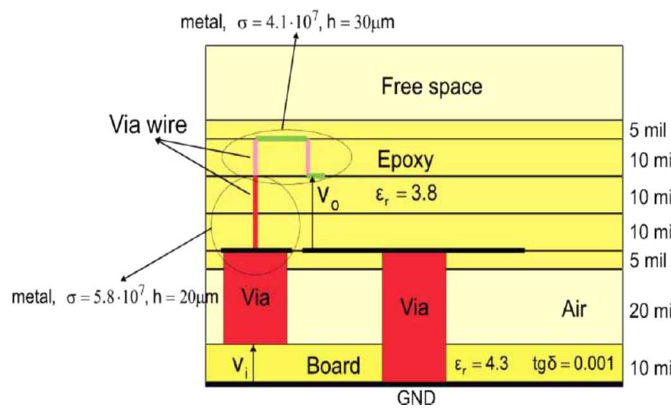

Fig. 2. BGA package: cross section of the structure [10].

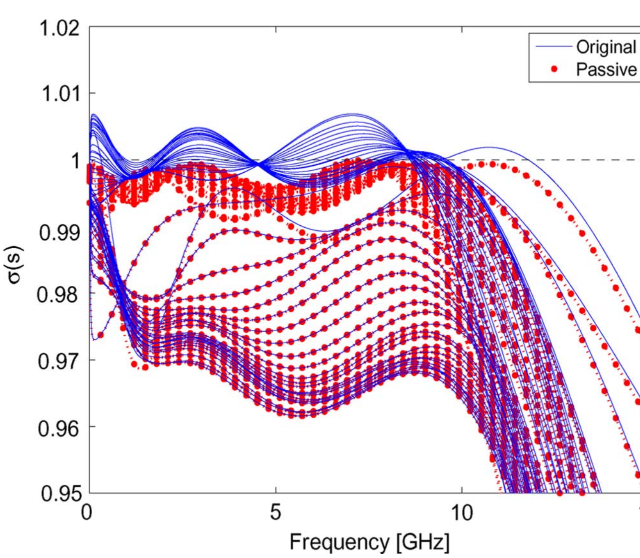

Fig. 3. BGA package: singular values of scattering matrix.

B. Adjustments of Residues

In order to make the initial state-space model passive, a new set of residues C_{viol} is computed by fitting the violation param-

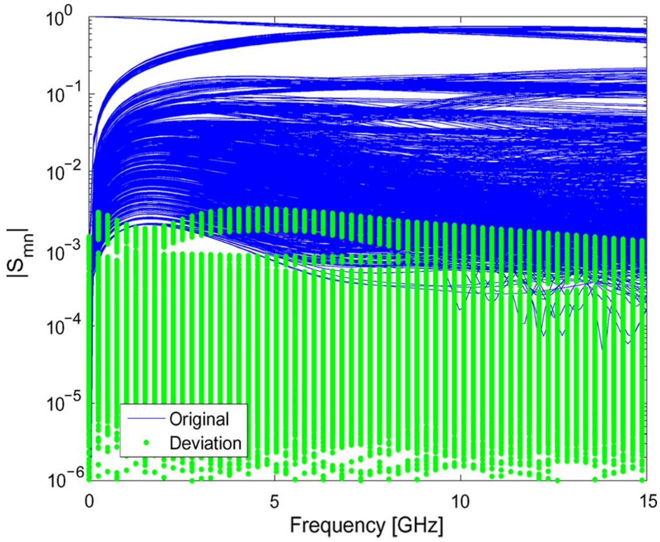

Fig. 4. BGA package: magnitude of matrix elements.

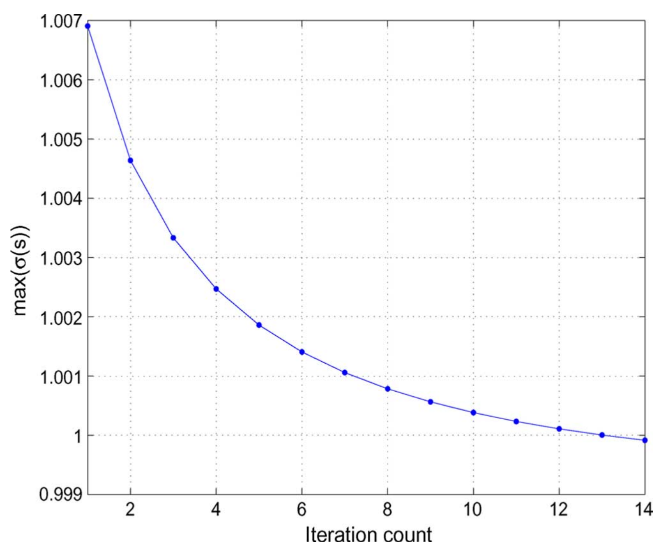

Fig. 5. BGA package: maximum singular value in each iteration step.

eters S_{viol} over the frequency sweep Ω_{eval} using the same set of poles A that were used in the original model (2)

$$S_{\mathrm{viol}}(j\omega) = C_{\mathrm{viol}}(j\omega I - A)^{-1}B. \tag{11}$$

It is noted that the solution of (11) is found by solving an overdetermined least squares matrix. The computational cost of this residue identification step is very small because it does not require any pole relocations. The calculated residues C_{viol} are then subtracted from the previous residue matrix C_t in order to suppress the passivity violations; hence,

$$C_{t+1} = C_t - C_{\mathrm{viol}}. \tag{12}$$

This process is repeated in an iterative way until all violations are compensated. The variable t is an index that denotes the tth step of the iteration process. An overview and flowchart of the passivity enforcement algorithm is shown in the Appendix.

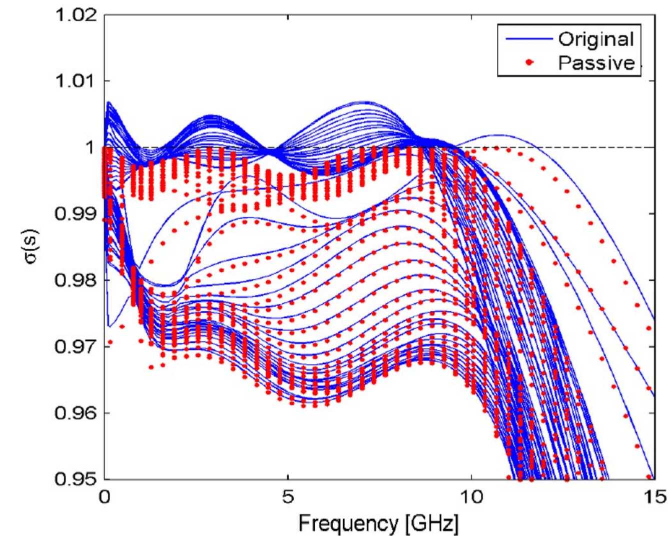

Fig. 6. Results Fig. 3 by Gustavsen's approach (absolute error control) [6].

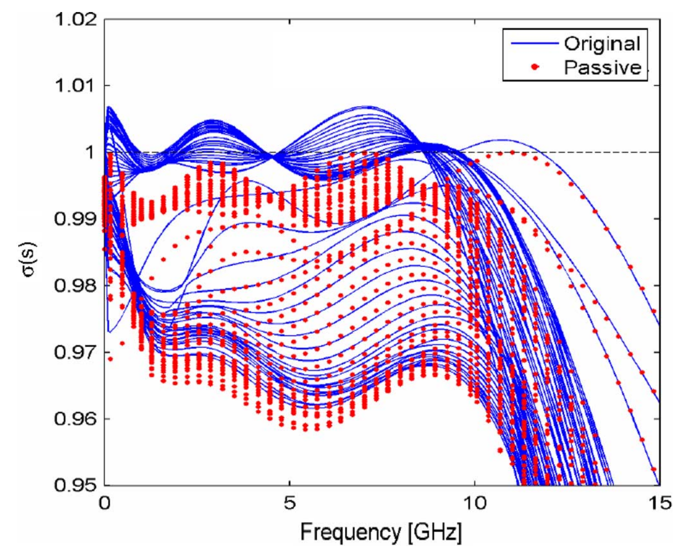

Fig. 7. Results Fig. 3 by Gustavsen's approach (relative error control) [6].

V. Example: BGA Package

In this example, the presented approach is used to compute a passive macromodel of a 48-port ball grid array (BGA) package, as reported in [10]. The top view and cross section of the structure are shown in Figs. 1 and 2, respectively. The scattering parameters of the structure are simulated with Agilent EEsof Momentum [25] from dc up to 10 GHz, and vector fitting is used to approximate the response by a six-pole proper transfer function using 100 data samples [6]. It is seen from Fig. 3 that the macromodel has several nonnegligle passivity violations, both inside and outside the frequency range of interest. The proposed passivity enforcement procedure is applied to compensate the violations, and converges to a passive macromodel in only 96 s on a Dual Core 2.4-GHz laptop computer. Fig. 4 shows that the accuracy of the overall macromodel is well preserved. The largest deviation that is introduced by the passivity enforcement algorithm over all matrix elements corresponds to −49.73 dB, which is quite small given the size of the maximum violation

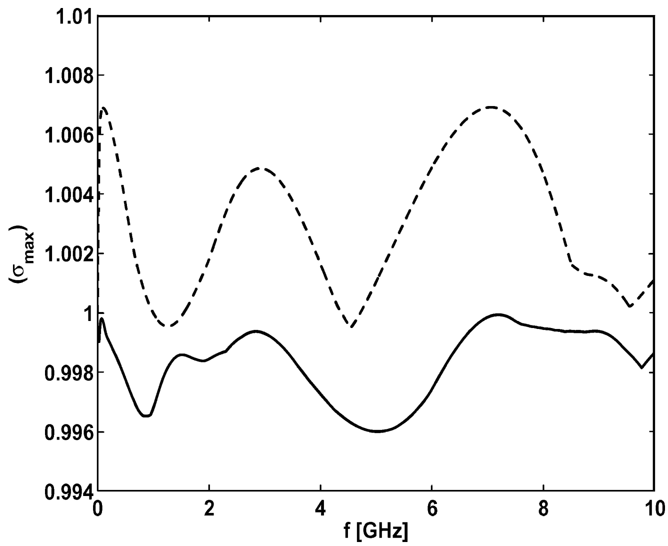

Fig. 8. Maximum singular value nonpassive (dashed line) and passive (solid line) model.

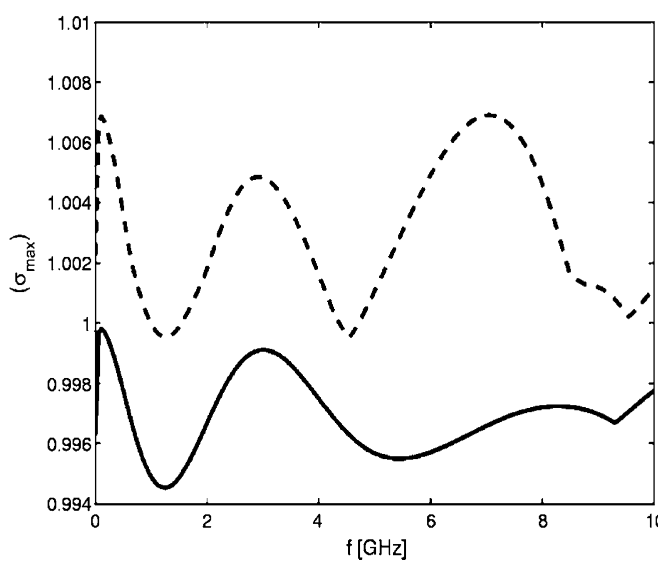

Fig. 9. Results Fig. 8 by Lamecki's approach (relative error control) [10].

($\sigma_{\max} = 1.0069$). Fig. 5 shows that the maximum singular value of the scattering matrix decreases monotonically in each iteration step. It is also observed that the proposed algorithm converges to a passive macromodel in only 14 iteration steps. The same BGA package was earlier used in [6] and [10] to demonstrate other passivity enforcement techniques, as shown in Figs. 6–9. In both cases, it is found that the deviation to the singular value curves is comparable or smaller using the new approach. Furthermore, this new method is much easier to implement by nonexperts in the field.

VI. Example: Interconnect System

In this example, the presented approach is used to compute a passive macromodel of a four-port chip-to-chip interconnect structure [26]. The test board with a solder-down transmitter and receiver packages is shown in Fig. 10. The scattering parameters of the structure are measured in the frequency domain

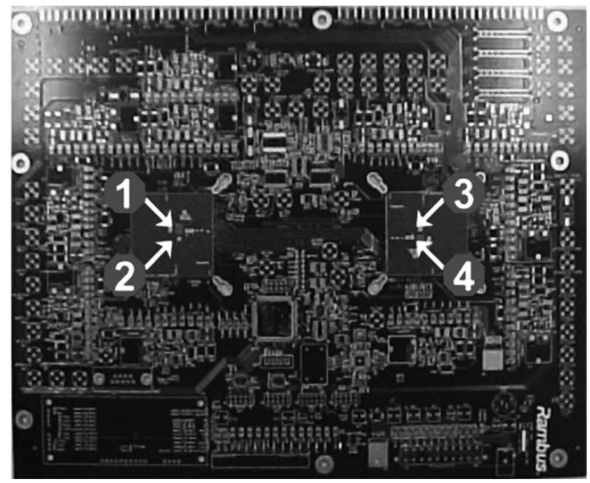

Fig. 10. Interconnect: overview of the test board with packages [26].

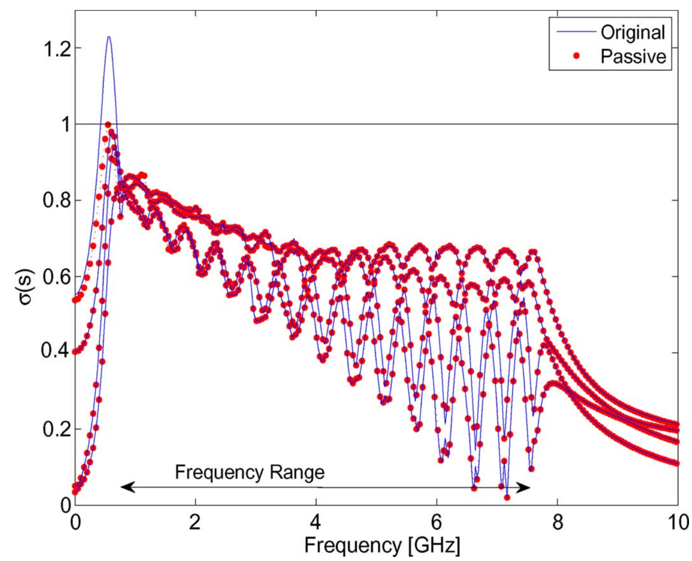

Fig. 11. Interconnect: singular values of scattering matrix.

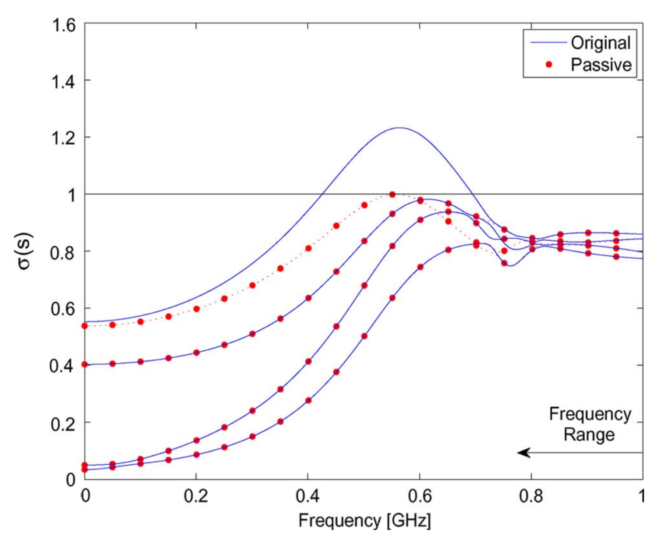

Fig. 12. Interconnect: singular values of scattering matrix at low frequencies.

from 775 MHz up to 7.52 GHz, and vector fitting is used to approximate the response by a 100-pole proper transfer function using 271 data samples [6]. It is seen from Figs. 11 and 12 that

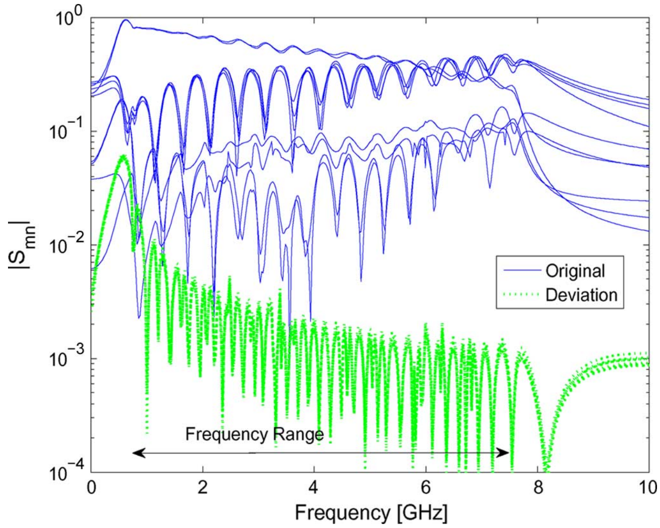

Fig. 13. Interconnect: magnitude of matrix elements.

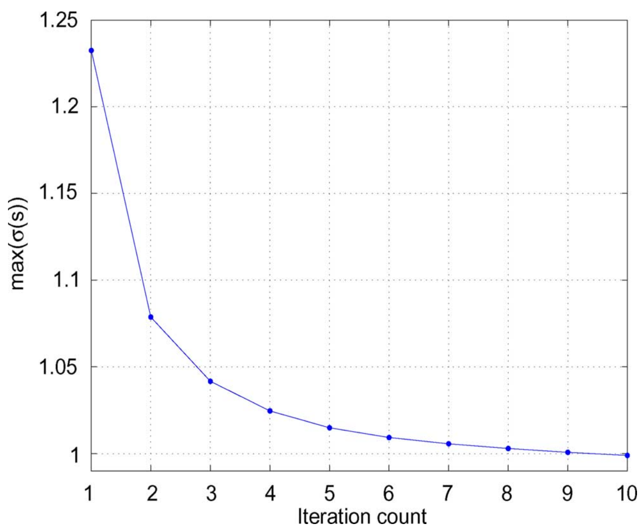

Fig. 14. Interconnect: maximum singular value in each iteration step.

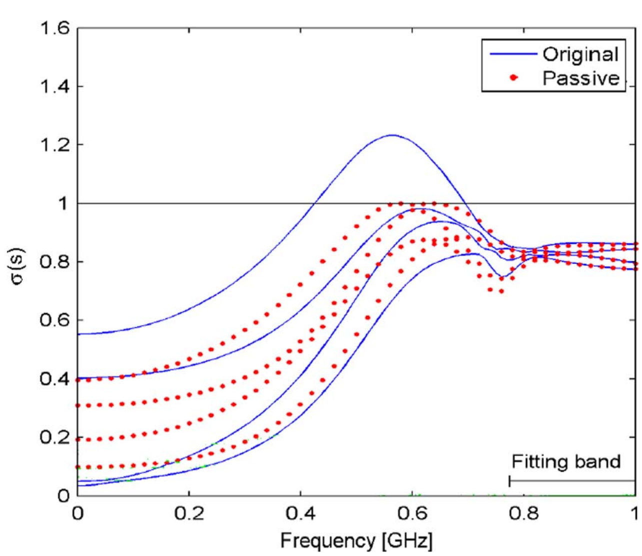

Fig. 15. Interconnect: results Fig. 12 by Gustavsen's approach [6].

the macromodel has a large out-of-band passivity violation at the lower frequencies. The passivity enforcement procedure is applied to compensate the violations, and converges to a passive macromodel in only 37 s on the same laptop computer. Fig. 13 shows that the accuracy of the macromodel is again well preserved. Fig. 14 shows that the maximum singular value of the scattering matrix decreases monotonically in each iteration step, and the algorithm converges to a passive model in ten iterations. The same interconnect structure was earlier used in [6] to demonstrate another passivity enforcement technique, and comparable results are obtained. It is shown in Fig. 15 that the new passivity scheme introduces a smaller deviation to the singular value curves at the lower frequencies (outside the frequency range of interest) since this part of the spectrum is also sampled in the frequency sweep Ω_{eval}.

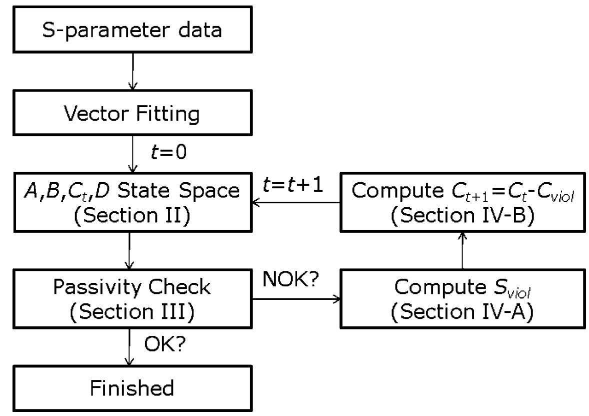

Fig. 16. Flowchart of the passivity enforcement algorithm.

VII. Conclusions

This paper has presented a novel technique for passivity enforcement of S-parameter-based macromodels, which does not require the use of optimization techniques. It iteratively computes updated values for the model residues until the singular values of the scattering matrix are unitary bounded. The implementation of the proposed algorithm is simple and straightforward. The robustness and efficiency of the method has been validated on a wide range of practical examples.

Appendix
Overview and Flowchart

Fig. 16 shows a flowchart of the passivity enforcement algorithm. Based on the measured or simulated scattering matrix of a linear structure, the vector fitting algorithm is applied to compute a state-space rational function approximation (Section II). A passivity test, based on the Hamiltonian matrix or on the eigenvalues of the transfer matrix at a limited number of relevant discrete frequencies, is used to verify if the macromodel is passive or not (Section III). If the model is passive, no compensation is necessary and the algorithm terminates. If the model is found to be nonpassive, the SVD algorithm is applied to calculate a set of violation parameters S_{viol} over a well-defined frequency sweep Ω_{eval} (Section IV-A). Consecutively, the residue identification algorithm is applied to find a new set of residues C_{viol} (Section IV-B). These residues are subtracted from the previous residues, and the iteration step t increases by one. This procedure is repeated in a recursive fashion until all passivity violations are collapsed, and a passive macromodel is obtained.

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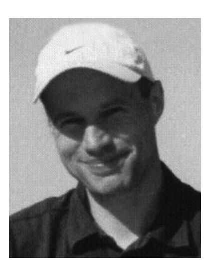

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