Accurate Passivity Enforcement Algorithm for Broadband S-Parameter Macromodels

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***Abstract*—This paper applies a reliable and efficient algorithm for passivity enforcement of multiport S-parameter macromodels. It minimizes the passivity violations in each iteration step, and converges to a uniformly passive macromodel. The method iteratively updates the residues of the macromodel by means of a simple least-squares fitting procedure. Numerical examples indicate that passive macromodels are obtained at a limited computational cost while preserving a good accuracy.**

I. Introduction

The synthesis of accurate broadband macromodels from tabulated S-parameter data is very important for the design of passive microwave systems and devices. Although standard identification techniques, such as e.g. Vector Fitting [1]-[4], are available to extract the model coefficients accurately, the resulting macromodel is stable but possibly non-passive. As a consequence, there has been a lot of attention in literature to solve this problem throughout the years [5]-[20]. This paper applies a new iterative passivity compensation algorithm [21] that is able to enforce passivity to a non-passive rational macromodel by means of a simple overdetermined least-squares fitting procedure. The main benefit of this approach is that it does not rely on optimization procedures which are often numerically expensive or possibly non-convergent. At the same time, the implementation of the proposed algorithm is simple and straightforward. The proposed method is applied to model a passive 48-port BGA package an a 4-port interconnect system. Some numerical results illustrate that this approach achieves an excellent trade-off between computation time and accuracy preservation of the overall macromodel.

II. Macromodeling

Vector fitting is an efficient macromodeling technique to compute a rational function approximation from the scattering matrix of a given linear structure [1]-[4]. A direct application of the algorithm to the simulated or measured frequency response yields a stable, but potentially non-passive macromodel that is formulated in a compact pole-residue form

$$S_{mn}(j\omega) = \sum_{p=1}^{P} \frac{c_p^{mn}}{j\omega - a_p^{mn}} + d^{mn} \tag{1}$$

where $S_{mn}(j\omega)$ represents the corresponding element on row m and column n of the scattering matrix. The poles a_p^{mn} and residues c_p^{mn} are real or come in complex conjugate pairs, while d^{mn} is a constant real term. All elements of the scattering matrix can be fitted with a common set of poles ($a_p^{mn} = a_p$) or a separate set of poles for each scattering element. A complex diagonalized state-space realization of the compound system can easily be derived as shown in [1], [22]

$$j\omega X(j\omega) = AX(j\omega) + BU(j\omega) \tag{2}$$
$$Y(j\omega) = CX(j\omega) + DU(j\omega). \tag{3}$$

It is ensured that all the poles of the macromodel are strictly stable, such that the eigenvalues of A have negative real parts [23]. Asymptotic passivity of the macromodel is also enforced.

III. Passivity Condition Check

The definition of passivity for S-parameter based macromodels in the frequency domain stipulates that all singular values σ of scattering matrix $S(j\omega)$ are unitary bounded [24]

$$(I - S^H(j\omega)S(j\omega)) \geq 0 \quad \forall\omega \tag{4}$$

which leads to the following equivalent expression

$$\max_{\omega} \sigma(S(j\omega)) \leq 1 \quad \forall\omega. \tag{5}$$

This condition can easily be verified algebraically by computing the eigenvalues of an associated Hamiltonian matrix [25]

$$H = \begin{bmatrix} A - BR^{-1}D^TC & -BR^{-1}B^T \\ C^TQ^{-1}C & -A^T + C^TDR^{-1}B^T \end{bmatrix} \tag{6}$$

where $R = D^TD - I$ and $Q = DD^T - I$. If $j\omega_k$ is an imaginary eigenvalue of H, then the corresponding frequency ω_k may denote the crossover between a passive and a non-passive frequency band [26]. By computing the slopes of the singular value curves at the purely imaginary eigenvalues, it is possible to pinpoint the exact boundaries of a passivity violation. If all the eigenvalues of H have a non-vanishing real part, then the system is passive. Proofs are given in [25].

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IV. Passivity Compensation

If the state-space model (2)-(3) is found to be non-passive by the Hamiltonian test (6), then a new passivity enforcement algorithm can be applied to compensate the violation. The presented approach iteratively updates the residues in the output matrix C_t (for $t = 0, ..., T$) by a simple least-squares fitting procedure until all passivity violations are removed. In the first iteration step $t = 0$ of the algorithm, $C_0 = C$ in (3).

A. Non-passive Residuals of Scattering Matrix

First, a dense set of frequencies Ω_{eval} is determined from DC up to about 20% above the highest relevant frequency. This highest relevant frequency is the maximum of the highest crossing from a non-passive to a passive region on one hand and the maximum frequency of interest on the other hand. For each frequency ω_{eval} in the set Ω_{eval}, a singular value decomposition of the scattering matrix is performed as follows

$$S(j\omega_{eval}) = D + C_t(j\omega_{eval}I - A)^{-1}B = U\Sigma V^*, \quad (7)$$

where Σ is a positive, real-valued diagonal matrix that contains the singular values, and U and V are unitary matrices. The inversion of the $(j\omega_{eval}I - A)$ in (7) is computationally fast, because it is a complex diagonal matrix. It is clear that one (or several) of the singular values in Σ will exceed unity, in the areas where the model is non-passive. Therefore, a new set of violation parameters S_{viol} is constructed as follows

$$S_{viol}(j\omega_{eval}) = U\Sigma_{viol}V^* \quad \forall \omega_{eval} \in \Omega_{eval} \quad (8)$$

with

$$\Sigma_{viol} = \Sigma\Upsilon - \Psi \quad (9)$$

where Υ and Ψ are square diagonal matrices

$$\begin{array}{ll} \Upsilon|_{ii,\Sigma_{ii}\leq\delta} = 0 & \Upsilon|_{ii,\Sigma_{ii}>\delta} = 1 \\ \Psi|_{ii,\Sigma_{ii}\leq\delta} = 0 & \Psi|_{ii,\Sigma_{ii}>\delta} = \delta \end{array}. \quad (10)$$

The value of δ is a predefined tolerance parameter that is chosen slightly smaller than 1 in practice (such as e.g. 0.999).

B. Adjustments of Residues

In order to make the initial state-space model passive, a new set of residues C_{viol} is computed by fitting the violation parameters S_{viol} over the frequency sweep Ω_{eval} using the same set of poles A that were used in the original model (2)

$$S_{viol}(j\omega) = C_{viol}(j\omega I - A)^{-1}B. \quad (11)$$

It is noted that the solution of (11) is found by solving an overdetermined least-squares matrix. The computational cost of this residue identification step is very small, because it does not require any pole relocations. The calculated residues C_{viol} are then subtracted from the previous residue matrix C_t in order to suppress the passivity violations, hence

$$C_{t+1} = C_t - C_{viol}. \quad (12)$$

This process is repeated in an iterative way until all violations are compensated. The variable t is an index that denotes the t-th step of the iteration process.

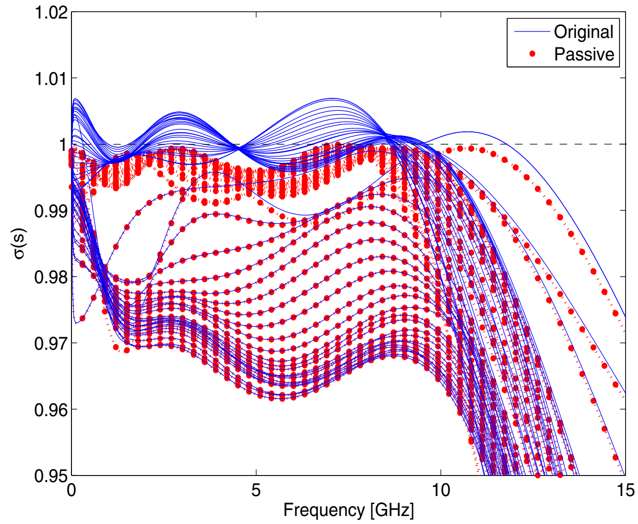

Fig. 1. BGA Package : Singular values of scattering matrix.

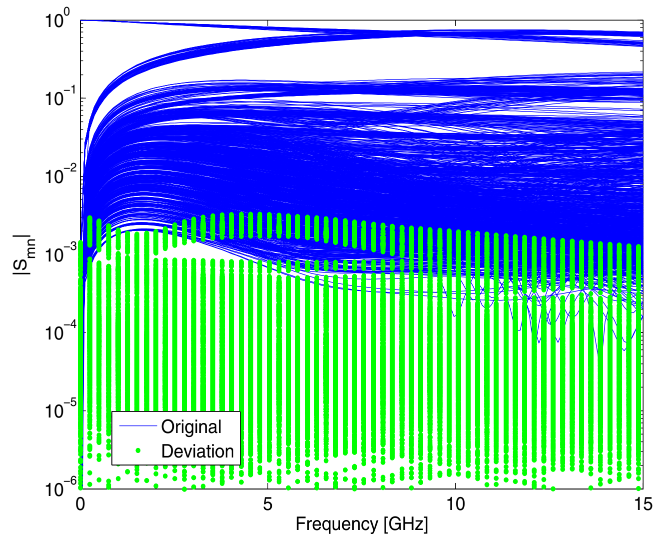

Fig. 2. BGA Package : Magnitude of matrix elements.

V. Example : BGA Package

In this example, the presented approach is used to compute a passive macromodel of a 48-port Ball Grid Array (BGA) package as reported in [12]. The scattering parameters of the structure are simulated with Agilent EEsof Momentum [28] from DC up to 10 GHz, and vector fitting is used to approximate the response by a 6-pole proper transfer function using 100 data samples [8]. It is seen from Fig. 1 that the macromodel has several non-negligle passivity violations, both inside and outside the frequency range of interest. The proposed passivity enforcement procedure is applied to compensate the violations, and converges to a passive macromodel in only 96 seconds on a Dual Core 2.4 GHz laptop computer. Fig. 2 shows that the accuracy of the overall macromodel is well preserved. The largest deviation that is introduced by the passivity enforcement algorithm over all matrix elements

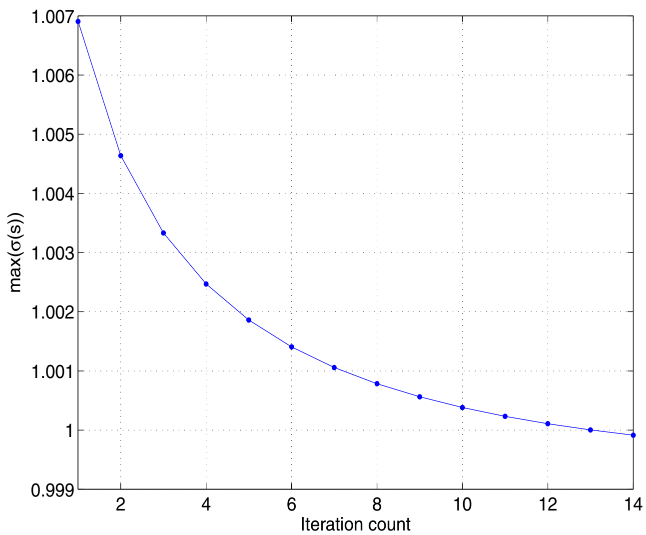

Fig. 3. BGA Package : Maximum singular value in each iteration step.

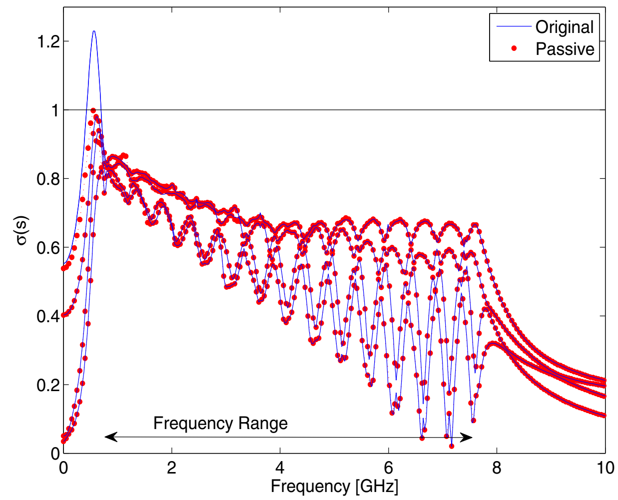

Fig. 4. Interconnect : Singular values of scattering matrix.

corresponds to -49.73 dB, which is quite small given the size of the maximum violation ($\sigma_{\max} = 1.0069$). Fig. 3 shows that the maximum singular value of the scattering matrix decreases monotonically in each iteration step. It is also observed that the proposed algorithm converges to a passive macromodel in only 14 iteration steps.

VI. Example : Interconnect System

In this example, the presented approach is used to compute a passive macromodel of a 4-port chip-to-chip interconnect structure [29]. The scattering parameters of the structure are measured in the frequency domain from 775 MHz up to 7.52 GHz, and vector fitting is used to approximate the response by a 100-pole proper transfer function using 271 data samples [8]. It is seen from Fig. 4 and Fig. 5 that the macromodel has a large out-of-band passivity violation at the lower frequencies. The passivity enforcement procedure is applied to compensate the violations, and converges to a passive macromodel in only 37 seconds on the same laptop computer. Fig. 6 shows that the accuracy of the macromodel is again well preserved. Fig. 7 shows that the maximum singular value of the scattering matrix decreases monotonically in each iteration step, and the algorithm converges to a passive model in 10 iterations.

VII. Example : Quarter Wavelength Filter

In this example, the presented approach is used to compute a passive macromodel based on simulated data of a 2-port quarter wavelength filter. The scattering parameters of the structure are simulated in the frequency domain from 1 GHz up to 12 GHz, and relaxed vector fitting is used to approximate the response by a 28-pole strictly proper transfer function using 1000 data samples. It is seen from Fig. 8 that the macromodel has several small passivity violations. The passivity enforcement procedure is applied to compensate them, and converges to a passive macromodel. The maximum singular value of the scattering matrix decreases monotonically in each iteration step. Fig. 9 shows that the accuracy of the macromodel is again well preserved.

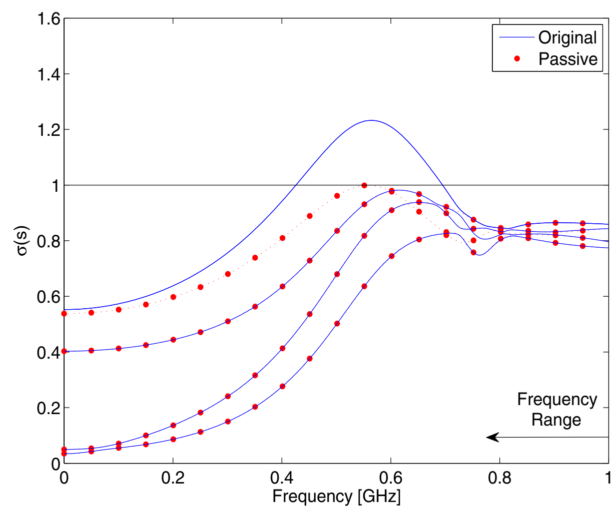

Fig. 5. Interconnect : Singular values of scattering matrix at low frequencies.

VIII. Conclusions

This paper applies a novel technique for passivity enforcement of S-parameter based macromodels, which does not require the use of optimization techniques. It iteratively computes updated values for the model residues until the singular values of the scattering matrix are unitary bounded. The implementation of the proposed algorithm is simple and straightforward. The robustness and efficiency of the method has been validated on a wide range of practical examples.

IX. Acknowledgments

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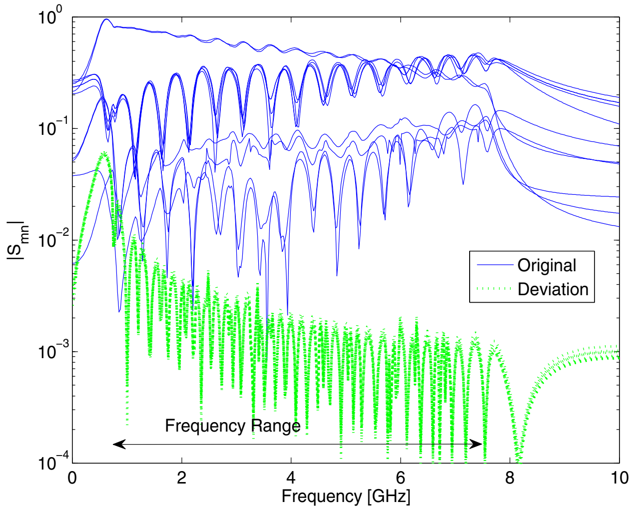

Fig. 6. Interconnect : Magnitude of matrix elements.

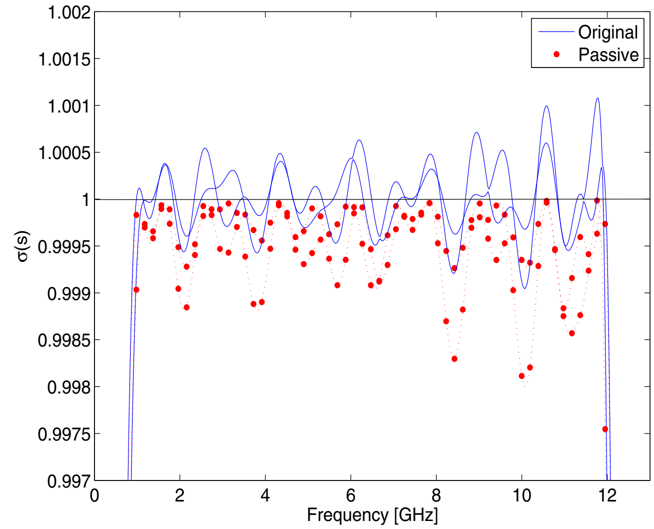

Fig. 8. Quarter Wavelength : Singular values of scattering matrix.

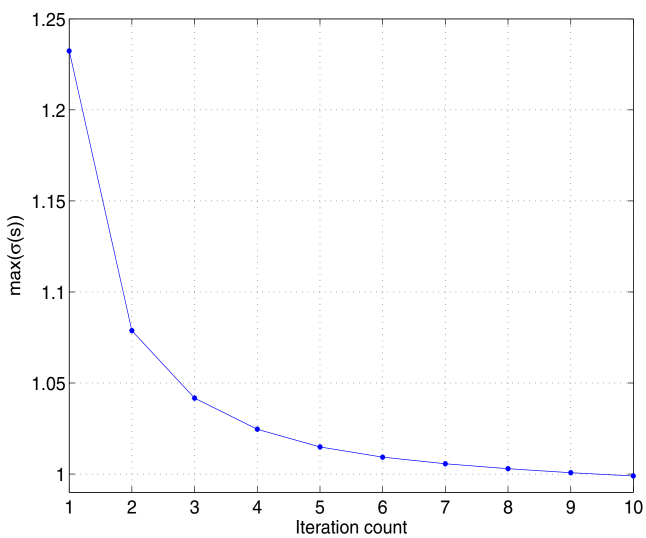

Fig. 7. Interconnect : Maximum singular value in each iteration step.

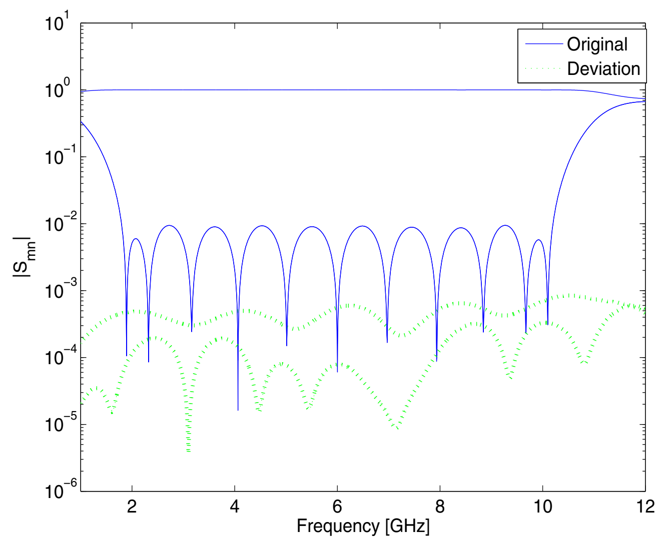

Fig. 9. Quarter Wavelength : Magnitude of matrix elements.

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