Numerical Validation of a Procedure for Direct Identification of Passive Linear Multiport with Convex Programming

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Abstract

The paper deals with the numerical validation, performance evaluation and robustness assessment of a procedure for the direct identification of passive transfer matrices based on convex programming. Validation is pursued by producing data sets from lumped multiport systems with random parameters (passive, non passive, and possibly affected by random noise), then evaluating the identification ability of the considered method. Results demonstrate how the considered approach satisfactorily covers the passive identification of a large class of data sets, even in presence of significant passivity violations or noise flawed data, at average accuracies comparable with non passive identifications obtained with standard Vector Fitting.

Introduction

A large number of electrical and electronic systems consist of linear distributed passive electromagnetic structures interacting with lumped elements (possibly non linear). Their system level analysis and design is largely based on circuit simulation. It is well known in literature how the passivity of the identified sub-systems should be guaranteed in order to get stable time domain system level simulations [1-3]. Also, some theoretical conditions are known for checking/enforcing passivity on rational models [4-6]. Several schemes have been presented for enforcing passivity "a-posteriori" with perturbation approaches, after a sufficiently accurate but non passive identification has been reached. Other schemes base on "a-priori" constraining passivity of the model during the identification process. A new procedure for identifying guaranteed passive models, originally named Positive Fraction Vector Fitting (PFVF), has been introduced in [7] (for the SISO case) and fully described (including generalization to MIMO systems) in [8], showing to work satisfactorily in practical cases of technical interest. Its formulation is briefly resumed in next section.

In this paper we validate numerically such passive identification scheme, with the analysis of a large number of frequency domain data for arbitrary lumped multiport systems. It is shown that the considered approach can be considered at present sufficiently robust in a large set of cases, while being accurate in the same order of standard "non passive" VF identifications.

A brief resume about the formulation

The "Positive Fraction" approach to passive identification of lumped multiport is based on three major facts:

1) a guaranteed passive system can be obtained as an expansion of N_p pole-residue passive terms as:

$$H(s) = R_0 + \sum_{n=1}^{N_p} \frac{R_n}{s - p_n} \tag{1}$$

where R_n are $M \cdot M$ (complex) residue matrices with M the number of ports;

2) passivity constraints on each pole residue term (or couple of conjugate terms) can be written in a frequency independent way as:

$$\begin{aligned} & R_0 \geq 0 \\ & R_n \geq 0 \text{ (real poles-residues terms)} \\ & \begin{cases} -(\mathrm{Re}\{p_n\}\mathrm{Re}\{R_n\} + \mathrm{Im}\{p_n\}\mathrm{Im}\{R_n\}) \geq 0 \\ -(\mathrm{Re}\{p_n\}\mathrm{Re}\{R_n\} - \mathrm{Im}\{p_n\}\mathrm{Im}\{R_n\}) \geq 0 \end{cases} \end{aligned} \tag{2}$$

where $\geq$ indicates "positive semi definite" in matrix context;

3) it is possible to identify residues as a convex optimization problem, after the poles have been accurately estimated by means of standard Vector Fitting (so that constraints 2 become "linear")

$$\{R_i\} = \arg\min_{\{R_i\}} \left\| vec\left(H(j\omega_k) - \tilde{H}(j\omega_k)\right)\right\|_2 , \tag{3}$$

where $\tilde{H}$ indicates the data.

After recognizing the problem as convex optimization, which guarantees a-priori the passivity constraint (2), a suitable software package can be used to solve it. In our procedure the solution is found with CVX software [9-10] which automatically transforms (3) with constraints (2) into semi-definite programming and calls the relevant solution routines.

General properties of the above formulation are fully analyzed in [8]. Here, we just mention the trade off between sub-optimality of the solutions, as compared to Positive Real Lemma based formulations, versus a significantly reduced

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computational cost. Moreover the expansion (1) leads directly to “concretely passive” synthesis schemes.

The validation scheme

Numerical validation of the considered procedure can be pursued by generating a general (lumped) multiport data set, eventually with intrinsic passivity violations and/or flawed by a certain level of random noise, to which we apply the passive identification scheme under investigation.

The data set generator is built on the basis of a general *M*-port synthesis scheme of dynamic order N_p, where random parameters are assigned (with possible passivity violations) in a physical way (by setting *R*, *L*, *C*, coefficients). Random noise is added at fixed % level after the describing frequency matrices are generated. Based on this data generation scheme, a large number of identifications have been carried out in order to validate the considered procedure with respect to:

- numerical stability, accuracy and convergence
- order reduction ability
- passivity violation recovery ability
- robustness in presence of noisy data

Although in order to fully evaluate the quality of a certain identification, a complete comparison of the data vs. fitted curve is necessary. A useful error index can be defined as:

$$err = \frac{1}{N_s}\sum_{k=1}^{N_s}\frac{\left\|Y_{data}\left(k\right)-Y_{fit}\left(k\right)\right\|_F}{\left\|Y_{data}\left(k\right)\right\|_F}, \tag{4}$$

where N_s is the total number of (frequency) samples and $\|\cdot\|_F$ indicates the Frobenius norm. Moreover, as long as the data and the VF identified model can present frequency intervals with passivity violations, it is useful defining the parameter ψ as the % of frequency intervals where violations occur referred to the overall interest frequency interval.

Some selected study cases

We report here a selection of cases showing the comparison of a standard VF identification (without any passivity enforcement) vs. the considered Positive Fraction Vector Fitting approach. This comparison is of conceptual more than practical importance, since in all a-posteriori enforcement technique there is some unavoidable degradation of accuracy in the final passive model as compared to the initial non passive VF identification.

case a): 2x2 120 poles passive data (with VF passivity violations).

The first example concerns a very common situation, where although the original data are passive, the VF identification lead to a non passive model. The example consist of a 120 poles passive two ports (its passivity can be checked by the eigenvalues test on matrix *Y* as shown in figure 1), identified with same number of poles. Identification results are shown in figure 2 where the standard VF identification and the PFVF identification of $\left|Y_{11}\left(\omega\right)\right|$ and $\left|Y_{12}\left(\omega\right)\right|$ elements are compared. First observe how, as it quite often happens, the VF identification introduces quite large number of passivity violations in the identified model, even in presence of purely

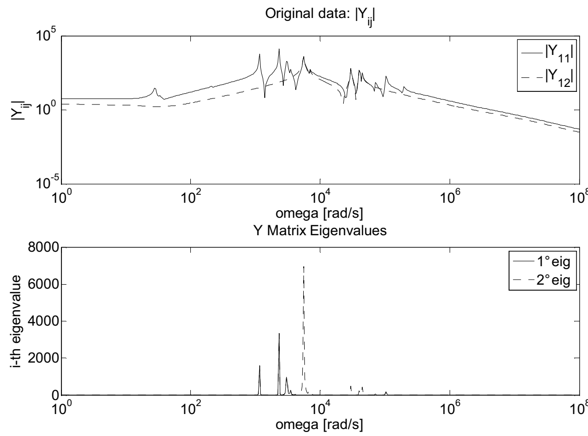

FIGURE 1: data and Re{$Y(\omega)$} eigenvalues for case a)

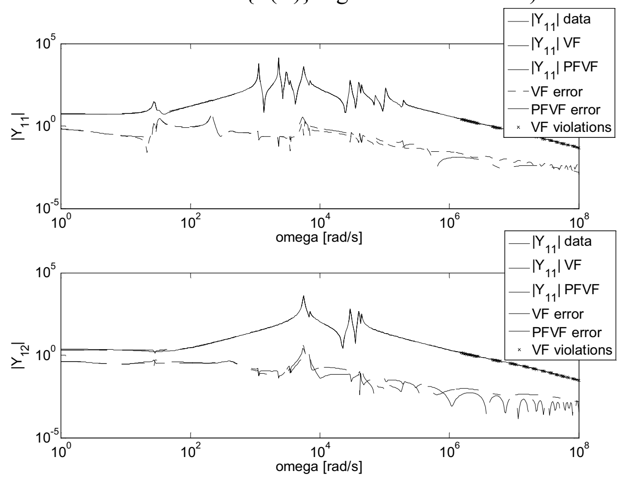

FIGURE 2: identifications of $\left|Y_{11}\left(\omega\right)\right|$ and $\left|Y_{12}\left(\omega\right)\right|$ for case a)

passive data without noise. The PFVF identification is able to fully recover this problem at substantially unchanged accuracy, as shown from the error curves. Moreover it can be noted that the PFVF error curve appear to be “smooth” as compared to the VF one in the regions where violations occur in VF identification. Main merit figures of this case, as for the following cases, are summarized in table I at the end of the section.

case b): 2x2 120 poles noisy data.

This example takes into account the presence of noise on the data set by adding a 2% random noise to a passive data set. Also in this case the VF identification introduces some passivity violations (see figure 3) which are easily recovered by the PFVF algorithm, with reasonably similar accuracy (see table I at the end of this section for details).

case c): 2x2 80 poles non passive data.

We consider now the case of non passive data. A 80 poles 2-port with data passivity violations range ψ_D =3 % of the entire frequency range. First note that VF identification broadens up to ψ_{VF} =35.6% the violation band amplitude (figure 4). The presence of violations in the data reflect also in the degradation of accuracy for VF as compared to previous cases. The accuracy achieved in both identifications is comparable, but favorable to PFVF (see table I for details).

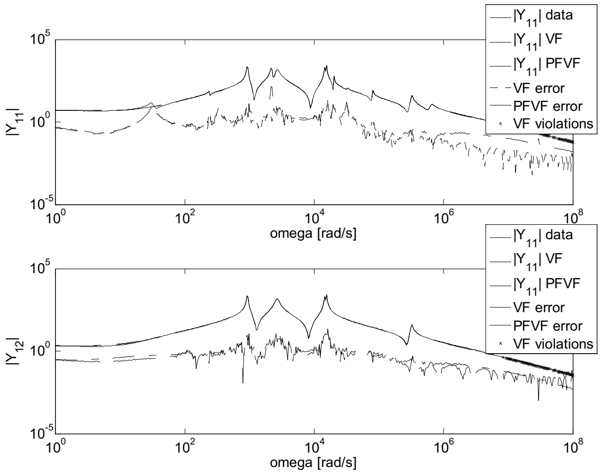

FIGURE 3: data and identifications for case b)

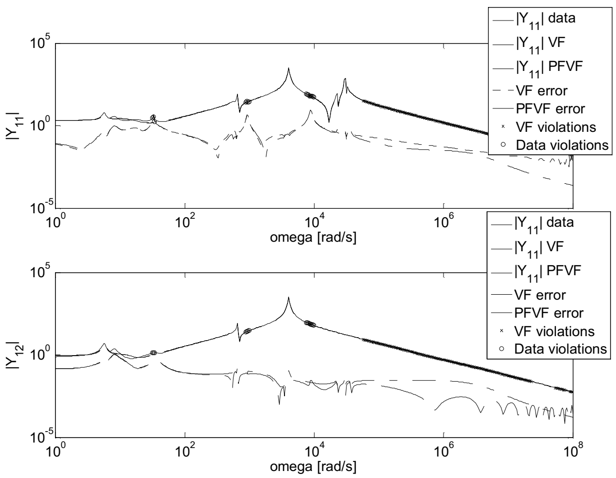

FIGURE 4: data and identifications for case c)

case d): 2x2 120 poles order reduction to 40 poles.

A further compared analysis has been carried out about the order reduction ability of the PFVF. The example we consider relates to noisy data (2%) from a 120 poles 2-port with no passivity violations, identified with 40 poles system both with VF and PFVF (figure 5). Both algorithms are able to fit the data with 40 poles at an acceptable accuracy, and the passivity violations introduced by VF are easily recovered by the PFVF. (see table I for details).

case e): 4x4x42 poles.

In order to evaluate the performances of the PFVF at increasing number of variables an example has been carried out for a 4x4 multiport with 44 poles with passivity violations in the data for a ψ_D=2% of the interest band. Results are shown in figure 6 and main figures in table I. It is apparent how the increased complexity does not change substantially the order of accuracy reached at comparable violation interval widths. (see table I for details).

Some statistical analysis

In this section we briefly report the results of a larger numbers of identifications to give some statistical insight on two issues concerning validation of the procedure, namely the data passivity violations recovery ability and the sensitivity to data noise.

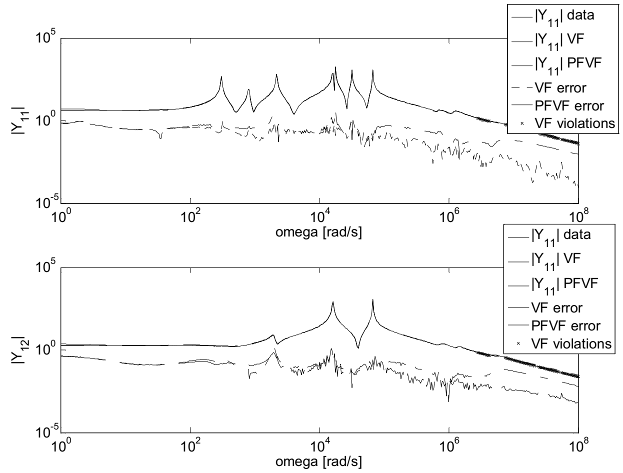

FIGURE 5: data and identifications for case d)

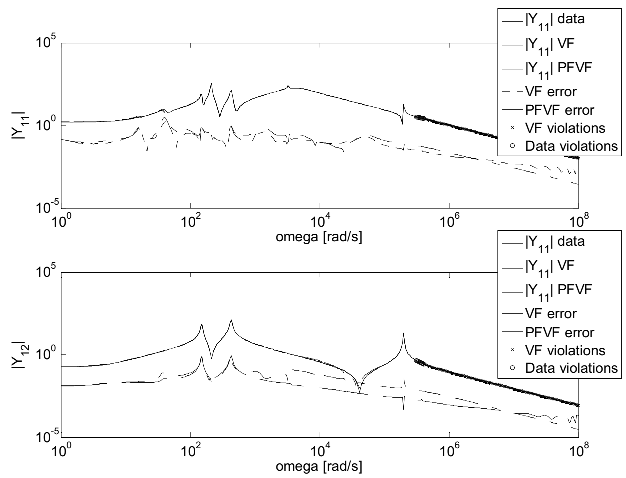

FIGURE 6: data and identifications for case e)

case	data poles	ident. poles	data noise	error VF	error PFVF	ψ=%viol. band Data	ψ=%viol. band VF
a)	120	120	0%	0.035	0.034	0	12.4
b)	120	120	2%	0.059	0.08	0	11.4
c)	80	80	0%	0.095	0.069	3.0	35.6
d)	120	40	1%	0.033	0.071	0	15.6
e)	44	44	0%	0.055	0.039	1.6	30.2

Table I - Major merit figures for the considered study cases

In order to investigate the first issue it is of some help to define some measure of the passivity violation "level", when occurring. As long as passivity violations can be detected, at each frequency ω, from the presence of negative eigenvalues in the $\mathrm{Re}\{Y(\omega)\}$ matrix, we define as a measure of the maximum violation level the index ν as:

$$\nu = \max\left(\frac{-\min(eig(\mathrm{Re}\{Y(\omega)\}))}{\max(eig(\mathrm{Re}\{Y(\omega)\}))}\right). \qquad (5)$$

After definition (5) we firstly show the comparison of results of 200 identifications of a 2x2-40 poles cases with passivity violations in (not noisy) data (figure 7). By considering the linear regressions it appears evident how the error in the identification of non passive data increases linearly (in logarithmic scale) with the violation level defined by (5). Moreover the PFVF error is on average better than VF identifications, this difference being increasing as function of violation level. A similar averaged behavior is obtained if the error is plotted vs. the parameter ψ_D previously defined.

The second issue, namely sensitivity to noise in data, has been investigated over a set of 240 identifications of a 2x2-40 poles cases, with increasing level of noise from 2.5% to 10%. The results are reported in figure 8. Here the performance of VF and PFVF, although similar, are in inverse order compared to the previous case. It can be argued that this result is due to the influence of noise on the VF system poles identification, that cannot be recovered by PFVF in the final residue identification step.

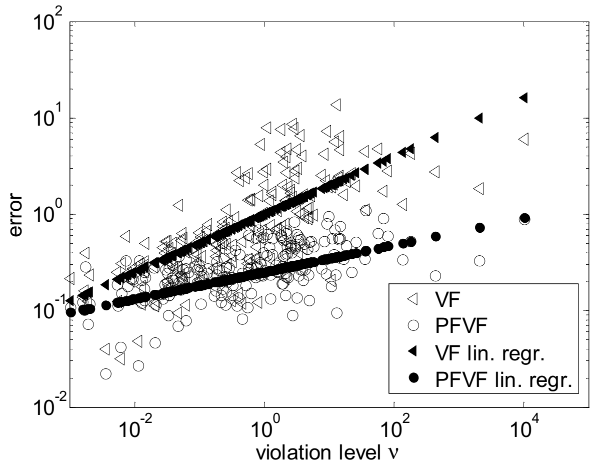

FIGURE 7: Identification accuracy as function of passivity violation level in data (with linear regression)

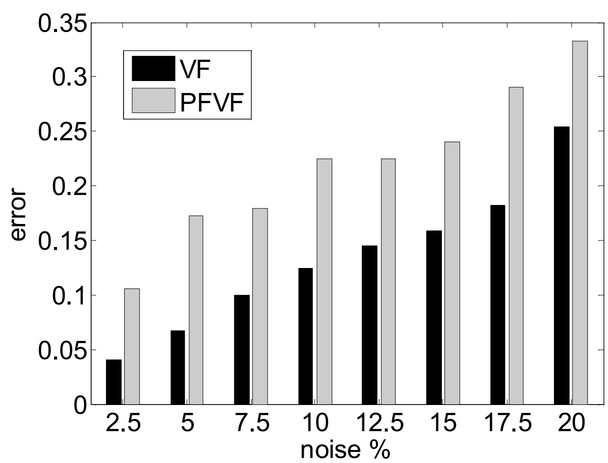

FIGURE 8: Average identification errors for increasing % noise levels on data

Conclusion and perspectives

The presented validation procedure shows firstly how the constraints on single terms in the expansion do not worsen (on average) the identification capability on passive lumped data set as compared to pure VF, which often give same identification accuracies but wit large violation intervals.

The identification of non passive data sets for significant violations (both as level and width of violation intervals) is very successful at high accuracies in a large number of data sets. On the other hand the procedure shows a moderate sensitivity to noise on data, higher on average than pure VF.

Although the convex optimization approach limits the practical use to moderate complexity problems (number of ports x number of poles), the Positive Fraction formulation reduces significantly the order of the problem as compared to Positive Real Lemma, giving at the same time advantages for concretely passive synthesis schemes.

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