# Modeling and Co-Simulation Framework for Multi-Wavelength Photonic Integrated Circuits: A Wideband Complex Vector Fitting Approach

Thijs Ullrick[1, 2], Dirk Deschrijver[1], Wim Bogaerts[2], and Tom Dhaene[1]

[1]Ghent University-imec, IDLab, Technologiepark-Zwijnaarde 126, 9052 Ghent, Belgium
[2]Ghent University-imec, Photonics Research Group, Technologiepark-Zwijnaarde 126, 9052 Ghent, Belgium

## ABSTRACT

Complex vector fitting (CVF), and in particular, its wideband extension, is a robust method to efficiently and accurately model the baseband scattering representations of linear and passive photonic integrated circuits (PICs). Such models allow for frequency and time-domain simulations at arbitrary optical carrier frequencies, making them especially well-suited for multi-channel system modeling and simulations. This paper presents a framework for intricate co-simulation of the passive and linear CVF models with non-linear active device models. To demonstrate the compatiblity of the proposed modeling framework with commercial photonic circuit simulators, a wavelength division multiplexing (WDM) system, comprising both active and passive devices, is simulated in Caphe, the circuit simulator of Luceda photonics.

## 1. INTRODUCTION

Recently, Complex Vector Fitting (CVF) has been proposed to efficiently and accurately model the scattering represenations of linear and passive photonic integrated circuits (PICs).[1] This technique, which computes baseband macromodels starting from the scattering parameters, can effectively capture complex photonic phenomena, including higher-order dispersion and wavelength dependent losses. Furthermore, to allow for time-domain characterization at arbitrary optical carrier frequencies, the wideband extension parameterizes these macromodels with respect to the optical carrier frequency, making them especially well-suited for the representation of multi-channel PICs.[2]

While the time-domain characterization of passive devices through the CVF modeling procedure has been extensively discussed, a methodology for the integration and simulation of these wideband models in commercial circuit simulators is still lacking from the literature. This work aims to bridge that gap. A framework is introduced for accurate and intricate co-simulation of the linear and passive wideband CVF macromodels with other passive or active devices. In particular, the wideband CVF macromodel is used to construct a multi-channel circuit model that can be seamlessly integrated in commercial photonic circuit simulators, and facilitates accurate co-simulation alongside both linear and non-linear behavioral models.

## 2. COMPLEX VECTOR FITTING

The CVF algorithm[1] starts from the scattering parameters of the photonic device under study in order to accurately capture non-ideal behaviours, such as higher-order dispersion, wavelength-dependent losses and backscattering. Let us assume that the scattering parameters of a photonic device have been acquired by means of EM simulations for a discrete set of frequencies within the bandwidth of interest: $\mathbf{S}(f_r)$ for $r = 1, ..., R$. The frequency response of the baseband equivalent system is then computed by shifting $\mathbf{S}(f_r)$ to baseband by substituting $f_i = f_r - f_c$, where $f_c$ is the optical carrier frequency. Next, the baseband scattering parameters $\mathbf{S}_l(f_i)$ are fed to the CVF algorithm, which builds a pole-residue model in the form[1]

$$\mathbf{S}_l(s) = \sum_{k=0}^{K-1} \frac{\mathbf{R}_k}{s - p_k} + \mathbf{D} \tag{1}$$

where $s = j2\pi f$ is the Laplace variable, $\mathbf{R}_k \in \mathbb{C}^{n\times n}$ are the computed complex residues, $p_k$ are the complex poles, and $\mathbf{D} \in \mathbb{R}^{n\times n}$ is a real matrix modeling the asymptotic response at high frequencies, where $n$ is the total number of ports of the system under study. Starting from the rational model (1), it is possible to analytically derive the corresponding system of *ordinary differential equations* (ODEs) in state-space form as[1]

$$\begin{cases} \frac{\partial \mathbf{x}_l(t)}{\partial t} = \mathbf{A}\mathbf{x}_l(t) + \mathbf{B}\mathbf{a}_l(t) \\ \mathbf{b}_l(t) = \mathbf{C}\mathbf{x}_l(t) + \mathbf{D}\mathbf{a}_l(t) \end{cases} \tag{2}$$

where $\mathbf{a}_l(t) \in \mathbb{C}^{n\times 1}$ and $\mathbf{b}_l(t) \in \mathbb{C}^{n\times 1}$ are the analytical forward and backward travelling waves of the $n$-port baseband system, corresponding to the RF modulated envelope of the photonic signal, $\mathbf{x}_l(t) \in \mathbb{C}^{m\times 1}$ with $m = nK$ represents the state-variables, $\mathbf{A} \in \mathbb{C}^{m\times m}$ is a diagonal matrix with $p_k$ at its non-zero entries, $\mathbf{B} \in \mathbb{C}^{m\times n}$ is a matrix that only has zeros or ones, $\mathbf{C} \in \mathbb{C}^{n\times m}$ is formed by horizontally stacking the residue matrices $\mathbf{R}_k$ and $\mathbf{D} \in \mathbb{R}^{n\times n}$ is the same matrix as in (1).

## 3. WIDEBAND COMPLEX VECTOR FITTING

To allow time-domain simulations at arbitrary wavelengths, the CVF macromodel is parameterized with respect to the optical carrier frequency by shifting its spectrum along the frequency axis by $\Delta f_c$.[2] The latter parameterization yields a new system of ODEs in the form

$$\begin{cases} \frac{\partial \mathbf{x}_l(t)}{\partial t} = (\mathbf{A} - j2\pi\Delta f_c \mathbf{I}_m)\mathbf{x}_l(t) + \mathbf{B}\mathbf{a}_l(t) \\ \mathbf{b}_l(t) = \mathbf{C}\mathbf{x}_l(t) + \mathbf{D}\mathbf{a}_l(t) \end{cases} \tag{3}$$

Expression (3) represents a new baseband equivalent system at center frequency $f_{cs} = f_c + \Delta f_c$ by means of the state-space matrices $(\mathbf{A} - j2\pi\Delta f_c \mathbf{I}_m)$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$. The new model can be directly obtained by shifting all the poles of the state-space model (2) computed at the optical carrier frequency $f_c$ by $j2\pi\Delta f_c$. It is important to note that by tuning the parameter $\Delta f_c$, the baseband carrier frequency $f_{cs} = f_c + \Delta f_c$ of the wideband model can be varied as needed, allowing for time-domain simulations at arbitrary optical carrier frequencies.

## 4. MULTI-CHANNEL MODELING FRAMEWORK

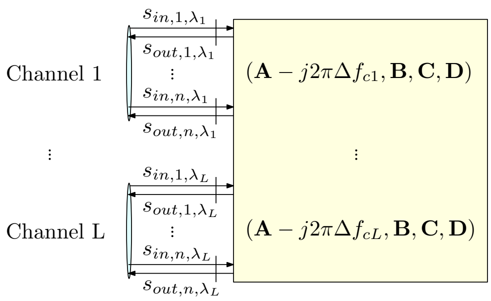


Figure 1. Multi-channel circuit models.

Accurate simulations of photonic circuits require models that can adequately describe both forward and backward scattering waves within the system. These waves represent the interactions and behaviors of optical signals as they propagate through various components, experiencing reflections, scattering, and other phenomena. The ability to account for both forward and backward waves is essential for capturing effects like back-reflections and backscattering, which can significantly impact the overall performance of photonic devices. Fortunately, many modern photonic circuit simulation tools such as Caphe[3] have incorporated support for these bidirectional wave interactions. Additionally, these software environments not only support forward and backward scattering waves but also provide built-in ordinary differential equation (ODE) solvers. This combination of features allows for the seamless integration and simulation of models described by the form (3).

The wideband macromodel offers the flexibility to be simulated at arbitrary optical carrier frequency by adjusting the parameter $\Delta f_c$ in (3). This parameterization with respect to the optical carrier frequency, is in fact an approach adopted by many circuit simulators. Caphe, for instance, allows to implement custom circuit models that are parameterized w.r.t. the optical carrier of the simulation. Multi-wavelength system analysis is then achieved by running various instances of the circuit, each representing the system at a different wavelength channel. The main advantage of this approach is that it allows for parallel simulation of the various circuit instances on multiple CPU cores, which can significantly reduce the CPU time when the number of wavelength channels is high. This approach is however only feasible when all devices within the circuit operate in the linear regime, ensuring that the output signals of each component associated with a particular wavelength are only affected by input signals from that same wavelength.

A more comprehensive approach, which is not constrained to the simulation of linear circuits, involves assigning input signals at different wavelengths to distinct ports of the circuit model. This means that the circuit model of an $n$ port device, supporting $L$ different wavelengths channels, will end up having $nL$ logical ports. In the particular case of the wideband CVF macromodeling framework, this is achieved by setting $\Delta f_c$ in (3) according to the various wavelength channels of the PIC and aggregating the resulting model instances into an overall circuit model with a port interface as illustrated in Fig. 1. By adopting this approach, it becomes possible to calculate the outputs of the non-linear components from the inputs at the various wavelength channels.

## 5. APPLICATION EXAMPLE

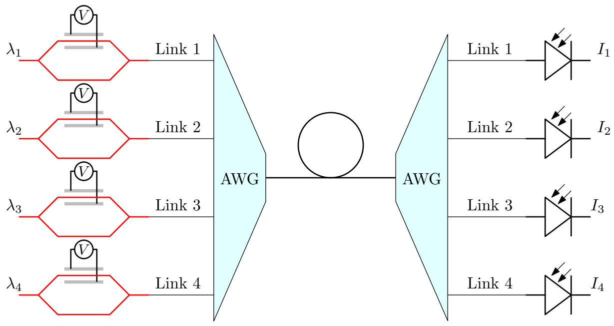


Figure 2. Functional diagram of the WDM system.

To demonstrate the compatibility of the proposed modeling framework with commercial photonic circuit simulators, the WDM system, illustrated in Fig. 2, is simulated in Caphe, the circuit simulator of Luceda photonics. Both the multiplexer and demultiplexer within the WDM system are implemented by the same 4-channel arrayed waveguide grating (AWG), as studied in the author's previous work.[2] The scattering parameters of the AWG, evaluated for 1001 uniformly distributed frequency samples, are shifted to baseband using $f_c = 194.51\,\text{THz}$. Following the CVF modeling procedure, a stable and passive CVF model is built with 26 poles, leading to a maximum modeling error between the data and the model response below $-61\,\text{dB}$. Next, the wideband CVF macromodel (3) is constructed and used to generate four model instances, each representing the AWG at its respective center wavelengths. The resulting state-space models are then integrated into a Caphe ODE model with a port interface compliant to Fig. 1. The modulators and photodetectors at the various wavelengths are modeled through simple analytic expressions that describe the bi-directional signal propagation within the device. The waveguide is characterized by its linear dispersion.

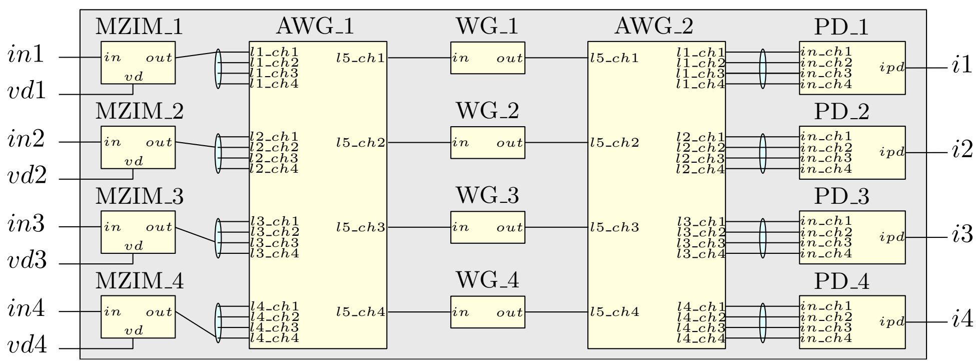


Figure 3. Schematic diagram of the WDM system.

Once all components are connected according the schematic diagram in Fig. 3, transient simulation of the WDM system is performed by applying four 10 Gb/s on-off keying (OOK) signals to the voltage inputs of the modulators. The optical inputs of the system on the other hand are excited with a constant signal, representing the output of a continuous wave laser at the respective center wavelengths of the AWG. The outputs of the modulator are subsequently multiplexed, sent over the waveguide, demultiplexed and

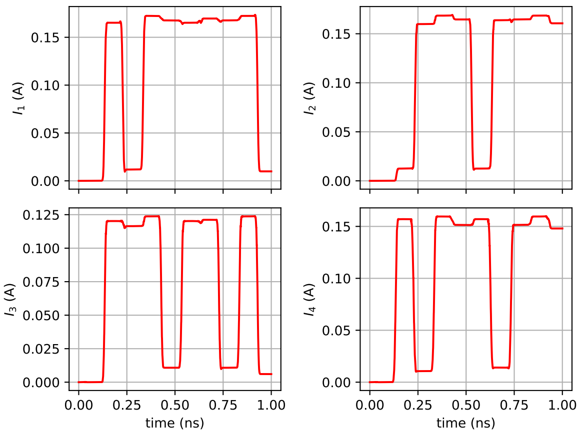


Figure 4. Transient output response of the WDM system simulated in Caphe.

detected by the photodiodes. The simulated photocurrents at the different output ports are illustrated in Fig. 4. The primary source of crosstalk in the AWG is imperfect imaging due to phase errors that arise from sidewall roughness: when the phases in the waveguides are not perfect, the image will be distorted and light gets coupled to other output waveguides. Another factor causing cross-talk is the limited bandwidth of the OOK signals, leading to energy leakage into adjacent wavelength channels. Consequently, these effects become evident in the form of four distorted OOK signals at the outputs of the WDM system. Despite the distortion, it is still possible though to decode the four bit sequences. The simulation of the WDM system consumed a total CPU time of 45.4 s. Given that the active devices are modeled by analytic expressions that can be quickly evaluated, it is reasonable to assume that most of that CPU time was allocated to numerically solve the various state-space models. It's also worth noting that Caphe employs a fourth-order Runge-Kutta solver, which, while versatile, is not the most efficient approach for solving a problem like (3).

Considering that all components of the WDM system are modeled in their linear regime, an alternative simulation approach is explored. Instead of assigning signals at different wavelengths to logical ports, distinct circuit instances are generated for each wavelength channel. By running these circuits on separate threads, it was possible to reduce the overall computational runtime to just 8.1 s. This experiment demonstrates the important role that parallel computing techniques can play in reducing the computational expenses associated with transient simulations of multi-wavelength systems.

## 6. CONCLUSION

The primary advantage of the proposed simulation framework lies in its ability to accurately and effectively model multi-channel dispersive devices, which can be simulated alongside both linear and non-linear circuit models. Furthermore, it is demonstrated that the proposed framework is compatible with commercial photonic circuit simulators such as Caphe. Finally, it is concluded that in order to mitigate the computational runtime when the number of wavelength channels is high, parallell computing schemes need to be explored and highly efficient numerical solvers will be required.

## REFERENCES


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